Timeless Partition-Strain on a Structured Algebraic Universe: A Two-Layer Regional-Disturbance Recovery Framework*

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Abstract

This paper develops a timeless partition-strain framework motivated by a basic tension in present foundations. General relativity and quantum mechanics both work with extraordinary precision, yet they organize reality through incompatible treatments of time, locality, and information. In quantum mechanics, time is normally treated as an external background parameter against which states evolve. In general relativity, time is part of the dynamical geometry itself, altered by gravitational structure. Quantum theory permits nonseparable correlations across distant systems, while relativistic gravity protects local causal structure through finite light-cone propagation. Black-hole evaporation sharpens the conflict: general relativity appears to allow information loss behind horizons, while quantum mechanics requires unitary recoverability.

The framework proposed here changes the starting point. Instead of treating spacetime, time evolution, gravity, quantum state update, and thermodynamic irreversibility as separate primitives, it begins from a tenseless algebraic substrate equipped with regional structure, conditional expectations, connection data, and a *paired regional disturbance bookkeeping* $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$. The first member is the universal record stiffness: the substrate's resistance to record-supporting deformations. The second member is the universal record susceptibility: the substrate's loading or distinguishability response under the same deformations. Both are positive operators, neither is defined from the other, and the observable record cone is controlled not by either operator separately but by the generalized spectrum

$$G_{\mathrm{acc}} = M_{\mathrm{rec}}^{-1/2} K_{\mathrm{rec}} M_{\mathrm{rec}}^{-1/2},$$

which is the geometry the framework requires to converge, under refinement, to the substrate's intrinsic access spectrum. Physical phenomena are then read through a second layer: a memory-bearing observer whose experience is determined by record-ordered access to the substrate. In this view, time is not a primitive coordinate of the universe. Time appears at the observer junction, where a nontracial observer representation cuts a tracial, timeless substrate and generates modular record flow.

The central object is a regional partition-strain functional measuring what an observer-access algebra fails to recover from the full substrate. Entropy is the scalar unrecoverability of this disturbance. Inertia is resistance to algebraic support deformation, read through the stiffness operator K_{rec}. Gravity-like response is variation of the same disturbance with respect to accessible connection or metric-like data. Record susceptibility, read through M_{rec}, sets the entropy/distinguishability cost of those deformations. Gauge and field equations are stationarity conditions of the same regional disturbance under admissible connection variations. The framework therefore seeks a simpler organization: not many disconnected physical mechanisms, but many observer-effective readouts of one underlying paired access-defect geometry.

Thermodynamics becomes the guiding analogy. The arrow of time is not imposed as a fundamental direction in the substrate. It is the experienced direction of increasing record restriction for a Type III observer reading a Type II-like reality. The past feels fixed because its records have already been compressed into stable but incomplete access-cuts; the underlying substrate information is no longer fully recoverable from the observer's present algebra. The future feels open because those cuts have not yet been made. Thus the Second Law is interpreted not as a primitive law of universal machinery, but as the geometric limit of localized ignorance: the monotone growth of unrecoverability along record-ordered access.

On this reading, the apparent conflicts between general relativity, quantum mechanics, thermodynamics, and information recovery are relocated. They are not treated as contradictions inside one spacetime ontology, but as different limits of substrate information seen through observer access. Local causal cones arise from finite record recoverability of the paired operator $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$, not from a primitive substrate speed limit. Quantum nonseparation reflects correlations in the substrate that need not be reducible to observer-local records. Horizon information loss becomes loss relative to an exterior access algebra, not destruction in the full substrate. Gravity becomes the macroscopic connection response of regional disturbance, rather than a separate force added to quantum theory.

The aim is not to claim a completed derivation of known physics, but to formulate a mathematically disciplined change of perspective: a timeless algebraic substrate with a paired stiffness–susceptibility bookkeeping, a record-bearing observer layer, and a junction where time, thermodynamics, locality, inertia, gravity-like response, and information loss appear as coupled consequences of partition strain and finite recoverability through the generalized record geometry G_{acc}.

Contents

1 Introduction

Standard physics writes its laws on a manifold. A spacetime is assumed, fields are placed on it, equations evolve those fields in a time coordinate, and observers measure what evolves. The tension this produces with background-independent quantum gravity is the long-discussed problem of time [1, 2, 3]. This paper takes a different organizational stance. It treats the totality of physical structure as a tenseless algebraic object and treats observers — and therefore time — as a separate layer that cuts that object and accumulates records.

The contribution is not predictive. It does not claim nature is this object, does not derive any established physical law from first principles, and offers no falsifiable consequence at variance with experiment. What it offers instead is a compressed bookkeeping principle. Familiar structures — locality, mass, inertia, gravitational response, gauge phase, time, motion, collapse, field dynamics, and thermality — are not introduced as independent modules. They are organized as readouts of a paired regional disturbance functional relative to observer-access algebras.

The framework rests on two ideas. The first is that physical structure can be represented as static compatibility data of an algebraic totality without reference to a clock. The second is that physical experience — what is felt as temporal order, the arrow of time, motion, and measurement — can be organized as features of a memory-bearing access structure that cuts the totality and accumulates records. The two ideas are modular. They become a unified reconstruction framework when applied together.

The compression principle is the central change of emphasis. The primitive object is not entropy, inertial mass, gravitational charge, or gauge energy separately. The primitive object is the *paired regional disturbance bookkeeping* $(K_{\rm rec}, M_{\rm rec})$. The first member, $K_{\rm rec}$, is the universal record stiffness; the second member, $M_{\rm rec}$, is the universal record susceptibility. The intrinsic regional disturbance is

$$\mathcal{K}_R(\rho, \nabla, g) \;=\; \tau[\mathfrak{D}_R(\rho)^* \, K_{\rm rec} \, \mathfrak{D}_R(\rho)] \,, \qquad \mathfrak{D}_R(\rho) \;=\; \rho_R - E_R(\rho),$$

weighted by the substrate stiffness, while the record-distinguishability geometry of the same defect is carried by M_{rec}. Entropy is the scalar unrecoverability of this disturbance. Inertia is the Hessian of this disturbance along algebraic support deformations, weighted by K_{rec}. Gravity-like response is the variation of this disturbance with respect to the accessible connection or metric-like embedding. Gauge and field equations are stationarity conditions of the same disturbance under internal connection variations. Crucially, the observable record cone is not controlled by K_{rec} or M_{rec} separately; it is controlled by the generalized record operator

$$G_{\mathrm{acc}} \;=\; M_{\mathrm{rec}}^{-1/2} K_{\mathrm{rec}} M_{\mathrm{rec}}^{-1/2},$$

whose spectrum the framework requires to converge, under refinement, to the substrate's intrinsic access spectrum. Thus the paper does not attempt to fit each phenomenon with a separate phenomenological term. It asks how many effective laws can be read from one paired regional defect geometry.

This is not a proposal to quantize gravity as a separate sector, nor to geometrize quantum mechanics by imposing a spacetime interpretation on operators. The statistical or quantum readout is the defect covariance, recovery loss, and modular state induced by a cut; the gravity-like readout is the connection or metric variation of the same disturbance; the gauge readout is its internal holonomy stationarity. Existing continuum theories remain reduction targets, but the gravity target is made explicit: the question is whether the spectral continuum limit of the generalized record operator G_{acc} yields an observer-effective action whose metric variation gives an Einstein tensor and whose defect variation gives a stress-energy tensor. In this sense quantum/statistical behavior and gravity-like behavior are not two primitive languages to be merged after the fact; they are two observer readouts of one paired trace-quadratic disturbance, one through defect covariance and state restriction, the other through connection and metric-like variation.

This paper makes three classes of claim. The first is a *substrate claim*: locality, non-separation, persistence, support-stiffness, gravity-like response, gauge phase, and record-stable cuts can be represented as relations among algebraic regions, conditional expectations, and the paired master disturbance functional. The second is a *reconstruction claim*: given a memory-bearing observer layer, familiar kinematic, quantum, gauge, and weak-gravity descriptions can be read as effective descriptions of recorded experience. The third is a *normal-form claim*: in a Type II_1 factor [4, 5, 7], the local disturbance associated with conditional-expectation defects has a forced quadratic leading term under explicit covariance and isotropy hypotheses, and the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ bookkeeping inherits the canonical trace through the generalized record operator.

The equivalence-principle content is therefore not a literal identity between two separately postulated forces. It is the claim that inertial resistance and gravity-like sourcing are two variational readouts of the same paired regional disturbance functional, evaluated along different observer directions. In weak, isotropic, low-disturbance limits these readouts reduce to the same trace-normalized quadratic coefficient, giving the observer-effective cancellation associated with composition-independent free fall. The same compression also fixes the interpretation of the observer speed bound: c_* is not a fitted number but a record-bandwidth threshold obtained from the lowest generalized eigenvalue of $G_{\mathrm{acc}} = M_{\mathrm{rec}}^{-1/2} K_{\mathrm{rec}} M_{\mathrm{rec}}^{-1/2}$.

The text proceeds as follows. Sections 2–3 define the substrate and observer layers and introduce the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ bookkeeping from the beginning. Section 4 recovers thermodynamics at the substrate–observer junction. Section 5 states the Shared-Connection principle as a shared-generator result. Section 6 fixes scope. Section 7 states the junction recovery principle for effective laws. Sections 7.1–7.6 give the observer-effective reconstructions: Newtonian mechanics, the finite record recoverability principle and observer-junction light limit, Lorentz kinematics, wavefunction and collapse, Maxwell-type equations, and weak-field gravity. Section 8 develops the algebraic necessity program, including the Type II_1 quadratic disturbance theorem, spectral covariance of defect modes from observer coarse-graining, the continuum Einstein-translation schema, disturbance-stationary actions, algebraic shell growth, modular observer access, modularly stable observer cuts, and non-Abelian selection. Section 9 formulates the Type $\mathrm{II}_1 \to$ Type III_1 observer junction as a modular embedding, including the conditional-expectation access cut, derivational support stiffness, recovery-loss susceptibility, and crossed-product observer core. Section 10 gives the framework-native reading of black-hole radiation. Section 11 tabulates the layer allocation and status of each claim. Section 12 presents the worked finite trace diagnostics. Section 13 runs a paired-pair audit on the universal $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ pair across record-cone, Maxwell-type stationarity, fine-structure, and lepton-ladder sectors at finite N. Section 14 compares the framework to neighboring foundation programs and Section 15 lists the remaining mathematical tasks. Section 18 develops the internal charged-sector identification and its observer-junction diagnostics built from the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ operator.

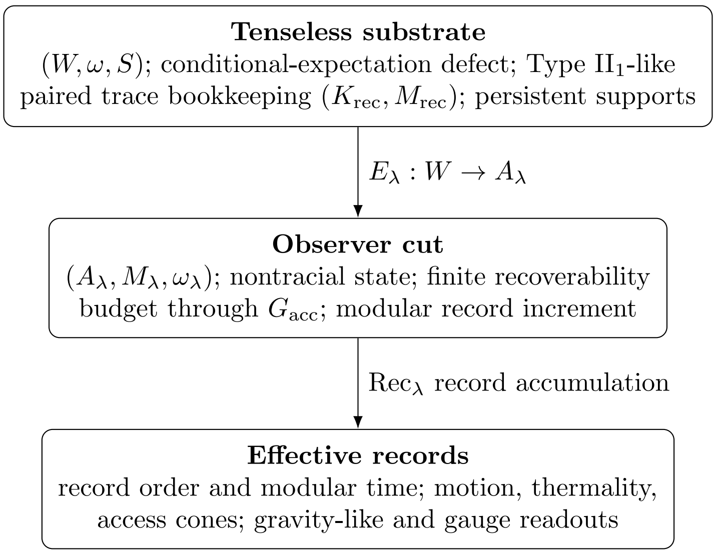

Figure 1: Layer map for the partition-strain framework. Static substrate data — including the paired stiffness–susceptibility bookkeeping $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ — define regional disturbance and persistent supports; a memory-bearing observer cut restricts that data; effective physics is reconstructed from stable, monotonically recoverable records.

2 The substrate layer

The substrate layer represents the structural side of physics. It proposes that what observers call locality, persistence, stiffness, curvature response, and gauge phase are features of a tenseless algebraic totality

$$\mathcal{S} \;=\; (W, \omega, S),$$

where W is a von Neumann algebra modeling all observables of the totality, ω is a faithful normal state, and S is declared additional structure — a regional decomposition, an admissible class of conditional expectations, an internal automorphism sector, and connection or metric-like data. Nothing in S refers to a clock or a direction of time.

Paired regional disturbance bookkeeping. The compressed substrate object is not a single stiffness or a single covariance. It is a paired bookkeeping

$$\boxed{(K_{\mathrm{rec}}, M_{\mathrm{rec}}) \;=\; (\text{record stiffness},\ \text{record susceptibility})}$$

both positive operators on the relevant defect Hilbert space, both trace-intrinsic, and *neither defined from the other*. The first member, K_{rec}, encodes the substrate's resistance to record-supporting deformations. The second member, M_{rec}, encodes the substrate's loading or entropy-distinguishability response to those same deformations. The observable geometry of the substrate is then carried by the generalized record operator

$$G_{\mathrm{acc}} \;:=\; M_{\mathrm{rec}}^{-1/2}\, K_{\mathrm{rec}}\, M_{\mathrm{rec}}^{-1/2},$$

which the framework requires to converge, under refinement of the substrate, to the intrinsic substrate access spectrum. This is a weaker compatibility condition than identifying K_{rec} with a function of M_{rec} or vice versa; it only requires

$$\mathrm{spec}\big(M_{\mathrm{rec}}^{-1/2} K_{\mathrm{rec}} M_{\mathrm{rec}}^{-1/2}\big) \;\to\; \mathrm{spec}(G_{\mathrm{acc}}),$$

without imposing a literal metric-compatible realization.

Structural decomposition of the paired operators. Both K_{rec} and M_{rec} admit a structural decomposition into substrate contributions. The susceptibility operator has the form

$$\boxed{M_{\mathrm{rec}} \;=\; M_0 \;+\; \eta_{\mathrm{acc}} M_{\mathrm{acc}} \;+\; \eta_{\mathrm{str}} M_{\mathrm{str}} \;+\; \eta_{\mathrm{ns}} M_{\mathrm{ns}} \;+\; \eta_{\mathrm{sec}} M_{\mathrm{sec}}}$$

and the stiffness operator has the matching form

$$\boxed{K_{\mathrm{rec}} \;=\; K_0 \;+\; \kappa_{\mathrm{acc}} K_{\mathrm{acc}} \;+\; \kappa_{\mathrm{str}} K_{\mathrm{str}} \;+\; \kappa_{\mathrm{ns}} K_{\mathrm{ns}} \;+\; \kappa_{\mathrm{sec}} K_{\mathrm{sec}}}$$

with the positivity requirements

$$M_{\mathrm{rec}} > 0, \qquad K_{\mathrm{rec}} > 0.$$

The ingredients have a uniform physical interpretation. On the susceptibility side, M_0 is a baseline record-loading metric (the irreducible record-distinguishability scale); M_{acc} is the record-loading contribution from access compliance, capturing how recoverable record information changes under the most basic access deformations; M_{str} is the susceptibility contribution from local substrate strain; M_{ns} is the susceptibility contribution from nonseparable record structure (record-loading carried by collective non-factorizing sectors); and M_{sec} is the susceptibility contribution from sector organization (charged, inertial, field-like, or entropy-carrying channels). On the stiffness side, K_0 is the irreducible record-stiffness baseline; K_{acc} is the access stiffness that resists changes in the accessible record configuration (in standard substrate models it specializes to the access Laplacian L_{acc}); K_{str} is the stiffness contribution from local substrate strain; K_{ns} is the stiffness contribution from nonseparable substrate structure; and K_{sec} is the stiffness contribution from sector organization. The coefficients $\eta_\bullet, \kappa_\bullet \geq 0$ are admissible weights; they determine the relative substrate-internal balance among the contributions and are not fitted from experiment.

The point of the decomposition is conceptual. It expresses that record stiffness and record susceptibility are not generic positive matrices to be chosen; they are built up from the same admissible substrate contributions (baseline, access, strain, nonseparable, sector), and each sector contributes independently to both members of the pair. In particular, $M_{\mathrm{rec}} \neq I$ in general, and $K_{\mathrm{rec}} \neq L_{\mathrm{acc}}$ in general: both operators are nontrivial, and the trivial normalization $(K_{\mathrm{rec}}, M_{\mathrm{rec}}) = (L_{\mathrm{acc}}, I)$ is a degenerate special case obtained by turning off every $\eta_\bullet, \kappa_\bullet$ except the access contributions and replacing the baseline susceptibility by the identity.

Regional defect and master disturbance. For an admissible observer-access region R, let

$$E_R : W \to A_R$$

be the conditional expectation [12, 13] onto the accessible regional algebra, and let ρ_R denote the regional representative of the state in a finite approximant or the corresponding state restriction in the algebraic setting. The regional defect is

$$\mathfrak{D}_R(\rho) \;=\; \rho_R \;-\; E_R(\rho),$$

understood abstractly as the part of the substrate state or support data that is not losslessly represented by the chosen regional algebra. The master disturbance functional is

$$\boxed{\mathcal{K}_R(\rho, \nabla, g) \;=\; \tau[\mathfrak{D}_R(\rho)^* \, K_{\rm rec}(\nabla, g, R) \, \mathfrak{D}_R(\rho)]\,.}$$

Here $K_{\rm rec}(\nabla, g, R)$ is the regional restriction of the universal record stiffness operator, τ is the canonical trace in the finite-factor approximation, and the paired susceptibility $M_{\rm rec}(R)$ enters through the recovery-loss / record-susceptibility functional of the same defect (made explicit in Section 7.1). In the purely algebraic setting the same formula denotes the trace-quadratic normal form of the defect geometry, and the paired operator $G_{\rm acc}$ becomes the controlling object for record propagation.

The named strain quantities are defined only as readout channels of $\mathcal{K}_R$. Factorization readout is the readout measuring failure of independent regional product structure. Coarse-graining readout is the readout measuring irrecoverability under E_R. Regional readout is the combined localization and access-defect readout over a partition. They are not independent primitive actions; they are projections, restrictions, or approximations of one regional disturbance bookkeeping.

None of these refers to a parameter that flows. Each is a static feature of the structure.

Substrate-level identifications. The substrate layer reads the following operational signatures as static features of the totality.

Space. The stable separability pattern of the substrate. When the regional disturbance has a deep, well-separated minimum at a partition Π^*, the regions of Π^* act as neighborhoods. Spatial proximity is small information distance under the disturbance geometry $G_{\rm acc}$; spatial extension is the number and relation of stable independent regions.

Mass. Persistent localized disturbance. A stable substrate excitation at a region R is an algebraic perturbation that persists under restricted automorphisms and increases $\mathcal{K}_R$ relative to a reference state. Its substrate-level mass readout is the local disturbance increment, weighted through $K_{\rm rec}$, not an independent primitive charge.

Inertia. Resistance to algebraic support deformation. A persistent excitation has a finite space of identity-preserving deformations of its relational support. These are not translations through pre-existing space; they are changes in which subalgebraic factors, joins, and support relations are included in the observer-access cut. The Hessian of $\mathcal{K}_R$ along this support-deformation manifold is the inertial readout, controlled by $K_{\rm rec}$.

Gravity-like response. Connection variation of the same disturbance. A substrate with a non-integrable connection sector exhibits loop holonomy whose response to localized disturbance is the substrate-level analog of gravitational sourcing. Variation of the paired stiffness–susceptibility geometry under connection deformations supplies the response channel.

Gauge phase. The holonomy of an internal automorphism sector. Local choices of phase under an internal symmetry can disagree across overlapping regions; the closed-loop mismatch is the gauge readout of the same connection-dependent disturbance geometry.

These are organizational identifications, not completed derivations. The substrate layer's proposal is that these operational signatures are read more cleanly as different faces of one tenseless paired regional disturbance $(K_{\rm rec}, M_{\rm rec})$ than as separately constructed phenomena.

3 The observer layer

The observer layer represents the experiential side of physics. It proposes that what observers call temporal order, the arrow of time, motion, and collapse are features of a memory-bearing access structure

$$\mathcal{O} \;=\; (\Lambda, \preceq, \{A_\lambda\}, \{M_\lambda\}),$$

where Λ is a directed set of access labels, $A_\lambda \subseteq W$ is the accessible subalgebra at label λ, and $M_\lambda \subseteq A_\lambda$ is the subalgebra of stable, decohered records at λ. The order $\preceq$ is derived from record-recoverability: $\lambda \preceq \mu$ when records at μ permit recovery of records at λ via a reverse channel.

Two primitive operations and one stability criterion. An observer performs two primitive access operations: cut and accumulate. Record persistence is not a third free operation; it is an admissibility criterion imposed by the paired substrate disturbance barrier.

Cut. At each label λ, the observer selects a conditional expectation $E_\lambda : W \to A_\lambda$. The substrate state ω is not modified by the cut; what changes is the observer's accessible marginal. In the strengthened junction formulation, the cut is not only a restriction of attention but the first map in a modular embedding:

$$(W, \tau, K_{\rm rec}, M_{\rm rec}) \xrightarrow{E_\lambda} (A_\lambda, \phi_\lambda) \xrightarrow{\pi_{\phi_\lambda}} M_\lambda^{\rm obs} := \pi_{\phi_\lambda}(A_\lambda)''.$$

The observer's effective time appears at this junction when ϕ_λ is faithful and nontracial, because the represented algebra then carries the modular automorphism group [9, 10] $\sigma_t^{\phi_\lambda}$. The substrate has not begun to evolve; rather, the observer has acquired a nontrivial modular flow on the algebra through which the substrate is represented.

Accumulate. The observer's record subalgebra grows along $\preceq$. Once acquired, records are not erased: $\lambda \preceq \mu$ implies $M_\lambda \subseteq M_\mu$ up to controlled error.

Local entropy-maximizing observer state. The observer cut is not an external agent placed on top of the substrate. For each access algebra A_λ, the observer-effective state is the least-biased state compatible with the records already stabilized in M_λ. In finite approximants one may write this as

$$\phi_\lambda \;=\; \arg \max_{\phi \in \mathcal{S}(A_\lambda)} \big\{ S(\phi) : \phi(m) = \omega(m) \text{ for all } m \in M_\lambda \big\},$$

with additional conserved record constraints included when present. Thus nontracial observer data are induced by partial access and record constraints, not introduced as an independent physical substance. If ϕ_λ is faithful and nontracial, its modular automorphism group supplies an intrinsic scale on the accessible algebra. The operational order, however, remains the inclusion order of stable records.

Inclusion index. The label λ is not primitive substrate time. It is an index of inclusion for stable access:

$$A_{\lambda_1} \subseteq A_{\lambda_2} \subseteq \cdots, \qquad M_{\lambda_1} \subseteq M_{\lambda_2} \subseteq \cdots,$$

up to the controlled recovery errors permitted by the observer budget. The word "later" therefore means: represented at a larger or more informative stable record algebra, not located at a later point in substrate time. When a faithful nontracial observer state is present, the associated modular automorphism parameter gives an intrinsic scale for this record order; the operational order itself is the inclusion chain.

Stability criterion. Record persistence is not left as a purely free observer postulate. A candidate record must be supported in a region whose erasure requires crossing a disturbance barrier generated by the paired $(K_{\text{rec}}, M_{\text{rec}})$ geometry. For a record support R, write

$$\text{Rec}(R) = 1 \iff \Delta\mathcal{K}_{\text{erase}}(R) \geq \Theta_{\text{rec}},$$

or, in a coarse threshold form,

$$\sigma_R^{\text{rec}} := \Phi_{\text{rec}}(\mathcal{K}_R) \geq \sigma_{\text{rec}}.$$

Here σ_R^{rec} is not an independent primitive functional; it is the record-stability readout of the paired master disturbance. The monotone record condition $M_\lambda \subseteq M_\mu$ is then interpreted as an observer-access rule restricted to high-barrier supports: records are stable because admissible low-disturbance updates cannot erase them without crossing the substrate stiffness threshold supplied by K_{rec} relative to the susceptibility cost supplied by M_{rec}.

Observer-level identifications. The observer layer reads the following phenomena as features of record-ordered access.

Time. The observer-effective parameter reconstructed from the inclusion order $\preceq$ of stable accessible records. Time is not a substrate coordinate in this framework; it is record order along an inclusion-indexed chain of cuts. When the local entropy-maximizing state ϕ_λ is faithful and nontracial, modular flow gives a canonical automorphism parameter for the accessible algebra; this modular parameter is a representation of observer-internal scale, while the operational arrow is the monotone inclusion of records. Different observers may therefore carry different access orders and different modular time scales.

Proper record time. When the observer-access structure is enriched by an invariant maximum access speed c_* derived from the generalized record operator G_{acc}, a stable excitation's support history also carries an observer-effective proper record time τ. This is not a substrate clock. It is a kinematic reconstruction rule on record-ordered support histories.

Arrow of time. The monotone growth of M_λ along $\preceq$. The arrow is not a substrate feature. It is the fact that records, once acquired, cannot be un-lived from inside the observer.

Motion. The variation of relational support of stable substrate excitations along $\preceq$.

Light cone. The boundary of A_λ at each λ. Operators outside A_λ are inaccessible at λ.

Collapse. Conditionalization inside A_λ. When a measurement returns a value, the observer's effective state on A_λ is the conditional. The substrate state ω on W is untouched.

Order and arrow. The substrate carries tenseless orderings on its admissible cuts and on its disturbance landscape. It carries no oriented arrow. A partial order is substrate-level and tenseless; an arrow is experiential and oriented. The observer does not create the substrate's orderings; the observer creates the experienced arrow by traversing those orderings while preserving records.

This is a locational claim, not by itself a derivation of thermodynamic irreversibility. The disturbance-barrier condition links observer memory to substrate stiffness, but thermodynamics only appears after the substrate and observer layers are combined. The next section makes that recovery explicit.

4 Recovered thermodynamics from substrate–observer coupling

Thermodynamics is not placed inside the substrate alone and is not assigned to an observer alone. The substrate supplies static regional disturbance, conditional-expectation defect, and the paired connection-dependent stiffness/susceptibility $(K_{\text{rec}}, M_{\text{rec}})$; the observer supplies cuts, accessible records, and an entropy gradient over those records. Only the pair

$$(W, \omega, S; \mathcal{O})$$

contains enough structure to define a thermodynamic exchange.

This separation is important. A bare substrate has no experienced heat flow, because it has no oriented record access. A bare observer cut has no physical content, because without substrate disturbance there is no cost whose variation can be read as flux. Thermodynamics is therefore recovered at the interface:

$$\text{regional disturbance flux} + \text{observer record entropy} \implies \text{thermodynamic balance.}$$

Cuts, entropy, and disturbance flux. Let $E_R : W \to A_R$ be a modularly stable observer cut, and let $\omega_R = \omega \circ E_R$ be the accessible state. The observer-side entropy of the cut is the entropy of accessible records. In finite approximants this is

$$S_R(\omega) = -\operatorname{Tr}(\rho_R \log \rho_R),$$

while in the von Neumann algebraic setting the intended replacement is the relative modular entropy associated with the restricted state and its reference cut. This entropy is not a substrate scalar by itself; it is the entropy of a state as viewed through an admissible access map, and its second variation along admissible deformations is the record susceptibility carried by M_{rec}.

The substrate-side quantity is the variation of $\mathcal{K}_R$ across the same cut. For a cut deformation, boundary support deformation, or connection variation ξ, define

$$\delta Q_R(\xi) \ := \ \delta_R \mathcal{K}_R(\rho, \nabla, g; \xi),$$

where δ_R denotes variation measured across the boundary between the accessible algebra A_R and its complement in the cut decomposition. In a boundary form this may be read as a disturbance flux

$$\mathcal{J}_{\partial R} \ \sim \ \langle \mathfrak{D}_R, \ K_{\text{rec}} \nabla_n \mathfrak{D}_R \rangle_{\partial R},$$

whenever an appropriate normal derivative or shell-gradient is available. The key point is that the heat-like quantity is not an additional source; it is the observer-accessible flux of the same paired regional disturbance.

Recovered Clausius relation. The modular data of (W, ω) supply the scale relating record-entropy change to access-flow change. Denote this scale by Θ_R. A cut is in local substrate–observer thermodynamic equilibrium when

$$\boxed{\delta Q_R \ = \ \Theta_R \, \delta S_R.}$$

This equation is not imposed at the substrate level. It is recovered only when disturbance flux through K_{rec} is paired with observer-access entropy whose Hessian is carried by M_{rec}. In this sense, thermodynamics is neither fundamental time nor merely psychological memory; it is the equilibrium bookkeeping of regional disturbance exchange across record-bearing cuts.

Relation to Jacobson. Jacobson's 1995 argument treats the Einstein equation as an equation of state by imposing $\delta Q = T\, dS$ on all local Rindler horizons, with horizon entropy proportional to area and Unruh temperature fixing the local temperature scale. The present framework keeps the structural moral but changes the primitive data. Jacobson begins with spacetime causal horizons; here the primitive object is a modularly stable algebraic cut. Jacobson's heat is energy flux through a horizon; here the corresponding flux is variation of the paired master disturbance across an observer cut. Jacobson's entropy is horizon entropy; here the entropy is accessible record entropy or its modular relative-entropy replacement.

The claim is therefore narrower than Jacobson's. This paper does not derive the Einstein equation. It proposes that gravity-like response becomes equation-of-state-like when the connection variation of one paired regional disturbance is read through record-bearing cuts. The inertia/gravity alignment of the next section is not itself thermodynamics; it supplies the shared generator whose variation becomes δQ_R once an observer cut is introduced.

Why the placement matters. The Shared-Connection principle can be stated as a substrate bookkeeping result: inertia and gravity-like response are not independent costs for the same paired regional disturbance. The thermodynamic relation needs more. It needs an observer cut, an entropy of accessible records, and modular stability of that cut. Therefore the logical order is

$$\begin{aligned}\text{paired regional disturbance } (K_{\text{rec}}, M_{\text{rec}}) &\Rightarrow \text{ inertial/gravity-like readouts} \\ &\Rightarrow \text{ observer cut } \Rightarrow \text{ thermodynamic balance.}\end{aligned}$$

5 Shared-Connection principle from one paired regional disturbance and Type II_1 trace uniqueness

The primary substrate-side route to the alignment of inertial response and gravity-like response is not a literal equality between two separately postulated forces. It is the compression of both readouts into one paired regional disturbance bookkeeping $(K_{\text{rec}}, M_{\text{rec}})$. The structural argument is the following.

The Type II_1 isotropic quadratic normal-form theorem of Section 8 says that, for Hilbert–Schmidt conditional-expectation defects, any positive, continuous, twice-differentiable, fully unitary-covariant disturbance functional has leading form

$$F(y) \;=\; c\,\tau_H(y^*y) \;+\; o(\|y\|^2_{\text{HS}}), \qquad c > 0,$$

where τ_H is the canonical trace on the Hilbert–Schmidt geometry of $L^2(M, \tau)$. The Murray–von Neumann uniqueness of the trace on a Type II_1 factor says that the substrate has no second canonical tracial weight from which to build a second independent leading defect geometry. Therefore any admissible readout that is a variation of the same paired regional disturbance inherits the same canonical quadratic bookkeeping, weighted through K_{rec} and measured by M_{rec}.

Proposition 5.1 (Shared generator for inertial and gravity-like readouts)**.** *Let (M, τ) be a Type II_1 factor and let X be a persistent excitation whose admissible support and connection variations are encoded by Hilbert–Schmidt defects satisfying the hypotheses of the Type II_1 normal-form theorem. Let*

$$\mathcal{K}_R(\rho, \nabla, g) \;=\; \tau[\mathfrak{D}_R(\rho)^*\, K_{\text{rec}}(\nabla, g, R)\, \mathfrak{D}_R(\rho)]$$

be the local regional disturbance. Define the inertial readout along an algebraic support-deformation direction v by

$$I_R(v) \;=\; \frac{d^2}{d\epsilon^2}\mathcal{K}_{R+\epsilon v}(\rho, \nabla, g)\bigg|_{\epsilon=0},$$

and the gravity-like readout along an accessible connection or metric-like variation h by

$$G_R(h) \;=\; \frac{d}{d\epsilon}\mathcal{K}_R(\rho, \nabla, g + \epsilon h)\bigg|_{\epsilon=0}.$$

Then I_R and G_R are not independent primitive couplings. They are variational projections of the same paired trace-quadratic disturbance geometry. In weak, isotropic, low-disturbance normalizations where the connection variation induced by an algebraic support deformation is represented by the same positive operator Δ_∇, both reduce to the same leading coefficient,

$$I_R(v) \;\simeq\; q_R^2\, v^T \Delta_\nabla v, \qquad G_R(v) \;\simeq\; q_R^2\, v^T \Delta_\nabla v,$$

with equality understood as a limiting shared-generator result, not as a primitive identity between separate forces.

Proof sketch. Both readouts are derivatives of one admissible local disturbance functional applied to defect variables generated by the same persistent support. By the normal-form theorem, the leading local geometry of those defects is anchored to the same canonical trace and hence to the same scalar coefficient c. Independent leading couplings would require independent canonical quadratic geometries on the same defect sector. A Type II_1 factor supplies only one faithful normal tracial state up to scale, and the full-covariance normal form supplies only one leading Hilbert–Schmidt defect norm. Thus the two readouts share a generator. In the weak isotropic limit the support and connection variations are represented through the same connection stiffness, giving the displayed common quadratic form. The paired susceptibility M_{rec} controls the same defect through entropy/distinguishability, so that the common eigenvalue of $G_{\mathrm{acc}} = M_{\mathrm{rec}}^{-1/2} K_{\mathrm{rec}} M_{\mathrm{rec}}^{-1/2}$ governs both readouts. □

This is the algebraic-substrate analog of the equivalence principle, not the full Lorentzian general-relativistic equivalence principle. In ordinary language, the equivalence principle says that inertial response and gravitational response cannot be operationally separated by the free fall of a test body. In the present framework that alignment is not introduced as a spacetime postulate. It is the observer-effective expression of a substrate bookkeeping fact: a Type II_1-like disturbance geometry supplies one canonical tracial quadratic coefficient for persistent regional defects, and the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ structure carries it consistently through the generalized record operator. To obtain the empirical free-fall statement, the observer-side recovery of support motion is still needed. That recovery is supplied in Section 7.1 as least-disturbance record continuation, and its weak-field version is supplied in Section 7.6.

Alternative motivation: support re-embedding as paired disturbance variation. A persistent excitation has one substrate cost for admissible re-embedding of its support,

$$C_X(v; \nabla) \;=\; \mathcal{K}_{R+v}(\rho, \nabla, g) - \mathcal{K}_R(\rho, \nabla, g).$$

If inertia and gravity-like response used different primitive functionals for the same support deformation, the identity of X would depend on which observer protocol was used. If the two operations merely commuted but remained distinct, the joined disturbance would still contain duplicate bookkeeping. Since the substrate has no primitive temporal order deciding which operation happens first, regional join disturbance must be path-independent:

$$\delta_I \delta_G \mathcal{K}_{R \vee S} \;=\; \delta_G \delta_I \mathcal{K}_{R \vee S}.$$

Minimal bookkeeping then collapses the two generators to one paired connection-dependent disturbance. The trace-uniqueness derivation above is the sharper mathematical form of this intuition.

5.1 Composition independence: the binding-energy question

The Shared-Connection principle is not strong enough if it applies only to a single elementary support. The empirical content of the equivalence principle is composition-insensitive: different bodies can contain different fractions of rest-like persistence, binding contribution, kinetic contribution, electromagnetic contribution, or other internal energy. A substrate account must therefore explain why internal contributions to a persistent composite do not carry independent gravity-like charges.

The framework's answer is that a composite excitation is still measured by the same paired master disturbance functional. Let a compound support in a region A be represented by persistent sub-supports $X_1, \ldots, X_k$. Its total substrate disturbance is one defect geometry,

$$\mathcal{K}_A(X_1, \ldots, X_k) \;=\; \tau[\mathfrak{D}_A^* \, K_{\mathrm{rec}} \, \mathfrak{D}_A] \,,$$

where the composite defect may decompose into physically labeled components. The observer's physical labels — rest-like, binding, kinetic, electromagnetic, gauge, or internal — describe different *components* of the defect, not different disturbance functionals. The Type II_1 normal form gives

$$\mathcal{K}_A \;=\; c\,\|\mathfrak{D}_A\|^2_{2,\tau,K_{\mathrm{rec}}} \;+\; o(\|\mathfrak{D}_A\|^2_{2,\tau}),$$

with the same c for all local defect contributions. Internal composition can change the magnitude and shape of the defect, but it cannot introduce separate canonical couplings c_{rest}, c_{bind}, or c_{em}. Such couplings would require separate canonical trace-quadratic geometries, which the finite-factor substrate does not supply.

A simple worked bookkeeping example makes the point explicit. Suppose the local defect associated with a composite support decomposes, relative to an observer's physical labels, as

$$\mathfrak{D}_X \;=\; \mathfrak{D}_{\mathrm{rest}} + \mathfrak{D}_{\mathrm{em}} + \mathfrak{D}_{\mathrm{bind}} + \mathfrak{D}_{\mathrm{kin}}.$$

Then the leading substrate disturbance is not a sum with independent gravitational weights. It is one paired Hilbert–Schmidt norm:

$$\mathcal{K}_X \;=\; c\|\mathfrak{D}_X\|^2_{2,\tau,K_{\mathrm{rec}}} + o(\|\mathfrak{D}_X\|^2_{2,\tau}),$$

so that, when the explicit stiffness weighting is locally suppressed from the notation,

$$\begin{aligned}\mathcal{K}_X/c \;&= \|\mathfrak{D}_{\mathrm{rest}}\|_2^2 + \|\mathfrak{D}_{\mathrm{em}}\|_2^2 + \|\mathfrak{D}_{\mathrm{bind}}\|_2^2 + \|\mathfrak{D}_{\mathrm{kin}}\|_2^2 \\ &\quad + 2\sum_{i<j} \Re\langle \mathfrak{D}_i, \mathfrak{D}_j\rangle_{2,\tau} + o(\|\mathfrak{D}_X\|_2^2).\end{aligned}$$

The cross terms matter because composition changes the defect shape. What is forbidden, under the one-trace normal form, is a replacement such as

$$c_{\mathrm{rest}}\|\mathfrak{D}_{\mathrm{rest}}\|_2^2 + c_{\mathrm{em}}\|\mathfrak{D}_{\mathrm{em}}\|_2^2 + c_{\mathrm{bind}}\|\mathfrak{D}_{\mathrm{bind}}\|_2^2,$$

with independently canonical coefficients. That expression would require separate preferred tracial quadratic geometries for different internal sources. The Type II_1 substrate supplies only the single Hilbert–Schmidt geometry induced by τ and the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ structure.

Thus the substrate-side equivalence statement is stronger than mass cancellation for an elementary support:

> all contributions to persistent regional disturbance couple through one leading paired defect geometry.

The observer-side reconstruction of trajectories is still needed to translate this into composition-independent recorded free fall.

6 Scope, modularity, and what the framework does not claim

The framework's claims sit in three classes.

Consistency claims. The substrate-side features and observer-side features named above can be implemented together on one unchanged finite substrate. Finite validations verify coexistence and bookkeeping consistency. They do not establish naturalness or uniqueness.

Reconstruction claims. Given the substrate-side structure $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ and the observer's access rules, the observer can reconstruct Newton-like mechanics, wavefunction-and-collapse, Maxwell-like equations, and a weak-field gravity response as effective descriptions of recorded experience.

Locational claims. Stable separability, persistence, stiffness, gravity-like response, and gauge connection are placed on the substrate side as readouts of paired regional disturbance. Time, the arrow, motion, collapse, and field evolution are placed on the observer side.

Scope of the claims. The framework should be read as a reconstruction program, not as a completed derivation of established physics. It studies the hypothesis that physical time, motion, thermodynamics, and field-like laws can be represented as observer-effective structures arising from access to a tenseless algebraic substrate. The paper does not assume as established fact that nature is such a substrate.

Several ingredients are specified constructively rather than derived uniquely from primitive axioms: the admissible regions, conditional expectations, connection sectors, source readouts, paired stiffness and susceptibility operators $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ and their structural decompositions, and observer-access rules. The compression introduced here reduces the number of independent disturbance readout sectors by treating them as readouts of one paired regional disturbance functional, but it does not yet derive that functional uniquely. The finite validations therefore demonstrate internal compatibility and feasibility, not naturalness, uniqueness, or empirical confirmation.

The recovered structures are structural analogues unless the additional continuum hypotheses stated later are satisfied. In particular, the paper does not claim an unconditional derivation of the Einstein field equations, the Standard Model, measured coupling constants, a physical continuum spacetime, or full diffeomorphism invariance. Section 8.3 gives the conditional continuum calculation required for the gravity claim: if the observer-limit action has an Einstein–Hilbert leading term and if the defect sector supplies a conserved stress tensor through metric variation, then the junction stationarity condition gives the Einstein equation. The coordinate-graph models used in the validation appendix are benchmarks, not substrate-intrinsic definitions. The dynamic metric-like field used there is a finite symmetric positive matrix field, not a Lorentzian metric.

Finally, "observer" does not mean a conscious agent. It denotes a memory-bearing access structure, in the operational sense used in quantum information and algebraic reconstruction.

Modularity. The two layers are modular. A reader interested only in the structural organization of physics can use the substrate layer without committing to the observer layer. A reader interested only in the foundations of temporal experience can use the observer layer without committing to the substrate layer. Each layer stands on its own. Together they yield a unified picture in which substrate phenomena and observer phenomena are explicitly separated and explicitly related, the bridge being the observer's cut and record accumulation.

7 Junction recovery principle for effective laws

The thermodynamic relation of Section 4 is recovered only after both layers are present. The same rule is used for all the other physical sectors in this paper. An effective law is counted as *recovered*, rather than merely stipulated as an observer readout, when it follows from three ingredients:

$$\text{paired regional disturbance} + \text{observer access} + \text{junction stationarity/compatibility}.$$

The substrate supplies tenseless objects: conditional-expectation defect, paired regional disturbance $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$, support stiffness through K_{rec}, record susceptibility through M_{rec}, connection response, holonomy, and algebraic shell growth. The observer supplies record order, accessible algebras, stable cuts, and finite or modular bandwidth. The junction principle is a balance, least-action, stationarity, conservation, or invariant-access condition that is not meaningful on either layer alone.

Thus the target is not to say that Newtonian mechanics, Maxwell equations, Lorentz kinematics, and weak gravity are hand-selected observer readouts. The target is to recover each as the effective equation forced when the observer is allowed to read one paired substrate disturbance through a stable access cut. The recoveries below are still limited: they do not establish standard physics in nature, do not determine numerical constants, and do not solve the Type III extension. But their logical status is stronger than a witness language. They are junction recoveries of the framework.

7.1 Recovered Newtonian mechanics as least-disturbance record continuation

Newtonian mechanics is recovered at the junction of support disturbance and record order. The substrate supplies a stable support manifold $\mathcal{S}_X$ for a persistent excitation X and a local disturbance functional $\mathcal{K}_X(q)$, where q^a are coordinates on the support-deformation manifold. These coordinates are not background space; they are parameters on the family of support embeddings accessible to the observer. The observer supplies a record chain $\lambda \mapsto A_\lambda$ and an internal label parameter $t = t(\lambda)$.

The junction principle is least-disturbance record continuation. Among support histories compatible with the observer's record chain, the realized effective history is stationary for

$$\mathcal{I}_X[q] \;=\; \int \Big(\tfrac{1}{2}\dot{q}^a K_{ab}(q)\dot{q}^b - \mathcal{K}_X(q) \Big)\, dt,$$

where

$$K_{ab}(q) \;=\; \frac{\partial^2 \mathcal{K}_X}{\partial q^a \partial q^b}$$

is the support-stiffness Hessian inherited from the paired master disturbance, weighted by K_{rec}. A record is unstable when the conditional expectation representing the observer's accessible memory fails to approximately preserve the previous support state under continuation; equivalently, the recovery error between successive observer cuts grows rather than remaining bounded. In this sense, a non-stationary path is not merely dynamically disfavored. It is a path along which the observer's algebra cannot maintain a stable conditional memory of the previous record.

The first variation gives

$$K_{ab}\, \ddot{q}^b + \Gamma_{abc}\, \dot{q}^b \dot{q}^c \;=\; -\partial_a \mathcal{K}_X,$$

with Γ_{abc} the connection induced by the support-stiffness metric when K_{ab} varies over the support manifold. In the local constant-stiffness limit this reduces to

$$K_{ab}\, \ddot{q}^b \;=\; -\partial_a \mathcal{K}_X.$$

Thus Newton-like mechanics is not substrate evolution. It is the observer's stable-record representation of stationary support continuation through a timeless paired disturbance landscape.

7.2 Finite record recoverability and the observer-junction light limit

Before introducing Lorentz kinematics, the speed limit must be located correctly. A tenseless substrate should not be assigned a primitive velocity bound by itself: velocity already presupposes both a distance and an oriented time parameter. The substrate supplies algebraic distance, disturbance barriers, modular stability, and conditional-expectation structure; the observer supplies record order, cuts, and an internal proper record increment. The finite light-like bound is therefore a substrate–observer junction quantity, not a substrate scalar alone.

Principle 7.1 (Finite Record Recoverability)**.** *A memory-bearing observer can recover new stable records only through admissible cut updates that preserve prior records, remain modularly stable, and do not cross forbidden disturbance-erasure barriers. The maximum rate at which newly recoverable stable records can enter the observer's accessible algebra defines the observer-junction speed.*

It is useful to distinguish this speed from a substrate propagation rate. If a chosen substrate flow or connection update admits a rate of disturbance spread, write it schematically as $v_{\rm sub}$. This is not yet the speed experienced by an observer, because propagation is weaker than record formation. A substrate disturbance becomes an event for an observer only if it passes three filters:

1. it is acquired into the accessible algebra A_λ;
2. it decoheres or stabilizes into the record algebra M_λ;
3. it remains monotonically recoverable along $\preceq$.

The observer-recoverability speed c_* is the maximal rate satisfying all three requirements. Consequently

$$c_* \;\le\; v_{\rm sub},$$

and the inequality can be strict.

Let $\mathcal{O} = (\Lambda, \preceq, \{A_\lambda\}, \{M_\lambda\})$ be an observer. The substrate supplies an intrinsic algebraic distance $d_\omega(R, S)$ between record supports, defined by the state and regional conditional-expectation structure rather than by coordinates. The observer supplies a proper record increment $\Delta\tau_\lambda > 0$ between adjacent labels. For a transition $\lambda \to \lambda^+$, define the admissible newly recoverable transitions by

$$\begin{aligned}\mathcal{R}_\lambda \;:=\; \big\{(R,S)\;:R \subset A_\lambda,\; S \subset A_{\lambda^+},\\ S \text{ contains a newly recoverable stable record caused by } R\big\},\end{aligned}$$

subject to the three record-recoverability constraints

$$S \subset A_{\lambda^+}, \quad S \subset M_{\lambda^+}, \quad M_\lambda \subseteq M_{\lambda^+},$$

together with modular stability and the disturbance-barrier condition

$$\|E_\lambda \sigma_t^\omega - \sigma_t^\omega E_\lambda\|_{2,\omega} \le \epsilon, \qquad \Delta\mathcal{K}_{\rm erase}(S) \ge \Theta_{\rm rec}.$$

The one-step record expansion is

$$\Delta_\lambda^{\rm rec} \;:=\; \sup_{(R,S)\in\mathcal{R}_\lambda} d_\omega(R,S),$$

and the observer-recoverability speed is

$$c_* \;:=\; c_{\mathcal{O}} \;:=\; \sup_\lambda \frac{\Delta_\lambda^{\rm rec}}{\Delta\tau_\lambda}.$$

The corresponding access cone is

$$d_\omega(R_\lambda, R_\mu) \;\le\; c_*\,|\tau_\mu - \tau_\lambda|.$$

This cone is not a statement that the substrate cannot contain nonlocal relations, nor that $v_{\rm sub}$ is the physical speed of light, nor that c_* has a universal numerical value in the present work. It is a statement that a record-bearing observer cannot make stable, ordered records recoverable faster than the admissible cut-update rate.

Equivalently, the finite value of c_* is an observer-junction recovery bound. Above this rate, the accessible correlation density of the substrate is too sparse, relative to the observer's modular record-time resolution, to support stable record reconstruction. It is therefore analogous to a Nyquist limit for observer records, not a primitive velocity imposed on the substrate. What is excluded is not substrate relation beyond this rate, but stable observer representation of such relation as an ordered recoverable record.

Paired record bandwidth from $(K_{\rm rec}, M_{\rm rec})$. The expected reduction form is the generalized eigenvalue of the paired stiffness–susceptibility operator. Let α_s be admissible algebraic deformations of the observer-access inclusion. These are not spatial translations in a pre-existing manifold; they are deformations of which regional factors, conditional expectations, and stable records are included in the cut. For a deformation direction v, the support-stiffness form is

$$\kappa_R(v,v) \;:=\; \frac{d^2}{ds^2}\mathcal{K}_{\alpha_s v(R)}\bigg|_{s=0} \;=\; \langle v, K_{\rm rec}\, v\rangle,$$

the mass-like side of the bandwidth law. It measures the quadratic disturbance cost required to deform the accessible support while preserving record identity. The record-susceptibility form is

$$\chi_R(v,v) \;:=\; \frac{d^2}{ds^2}D\big(\phi_{\alpha_s v(R)}\|\phi_R\big)\bigg|_{s=0} \;=\; \langle v, M_{\rm rec}\, v\rangle,$$

the entropy-like side. It measures how rapidly recoverable record information changes under the same algebraic deformation. In the sharper recovery version, χ_R is not merely an abstract entropy curvature. If $\mathcal{R}_R : A_R \to W$ is an admissible recovery channel associated with the cut, the junction recovery loss is

$$\mathcal{L}_R(\rho) \;:=\; S(\rho \,\|\, \mathcal{R}_R E_R(\rho)),$$

whenever the relative entropy is finite. For a deformation $\rho_s = \alpha_s(\rho)$ generated by an admissible derivation, one may take

$$\chi_R(v,v) \;=\; \frac{d^2}{ds^2}\mathcal{L}_R(\rho_s)\bigg|_{s=0}$$

up to the normalization used for modular record time. Thus record susceptibility is the Hessian of observer reconstruction loss. The zero-loss case is a cut whose accessible algebra recovers the relevant substrate state; positive loss is scalar unrecoverability. The record-speed scale is therefore not introduced independently; it is determined by the paired $(K_{\rm rec}, M_{\rm rec})$ structure.

Proposition 7.2 (Record-bandwidth bound from the paired generator)**.** *Let R be an observer-access region with paired master disturbance operators $(K_{\rm rec}, M_{\rm rec})$, both positive. A deformation mode v propagated at inclusion rate r is stably recordable only if*

$$r^2\,\chi_R(v,v) \;\le\; \kappa_R(v,v), \quad \textit{i.e.,} \quad r^2\,\langle v, M_{\rm rec}\, v\rangle \;\le\; \langle v, K_{\rm rec}\, v\rangle.$$

Consequently the local observer record-bandwidth bound is the generalized eigenvalue threshold

$$\boxed{\,c_*^2(R) \;:=\; \inf_{v\neq 0}\frac{\langle v, K_{\rm rec}\, v\rangle}{\langle v, M_{\rm rec}\, v\rangle} \;=\; \lambda_{\min}\big(M_{\rm rec}^{-1/2}\,K_{\rm rec}\,M_{\rm rec}^{-1/2}\big) \;=\; \lambda_{\min}(G_{\rm acc}),\,}$$

with the infimum taken over modes that remain in the stable record sector. Equivalently, stable modes solve the generalized eigenvalue problem

$$K_{\rm rec}v \;=\; c^2\,M_{\rm rec}v,$$

and the observer cone is the recoverable part of this generalized spectrum.

Proof. A deformation at inclusion rate r changes the observer state by an amount whose second-order distinguishability cost is $r^2\chi_R(v,v)$ per unit record increment. The same mode is stabilized by the disturbance stiffness $\kappa_R(v,v)$ supplied by the substrate. If the distinguishability growth exceeds the available stiffness, the reverse channel cannot keep previous records approximately recoverable and the deformation is not a stable element of the record algebra. Thus stable recordability requires $r^2\chi_R(v,v) \le \kappa_R(v,v)$ for every admissible mode. Solving for the largest uniformly admissible rate gives the Rayleigh-quotient bound above. The Rayleigh quotient $\langle v, K_{\rm rec}v\rangle/\langle v, M_{\rm rec}v\rangle$ equals the spectrum of $M_{\rm rec}^{-1/2}K_{\rm rec}M_{\rm rec}^{-1/2} = G_{\rm acc}$ since positive $M_{\rm rec}$ admits a symmetric square root, and the minimum of the quotient is the smallest eigenvalue of $G_{\rm acc}$. □

This proposition is the framework-native version of the relation between mass and entropy in the access speed. Mass-like inertia is not inserted as a separate constant; it is the stiffness Hessian carried by $K_{\rm rec}$ of the same paired regional disturbance. Entropy is not an independent thermodynamic add-on; it is the susceptibility of record distinguishability carried by $M_{\rm rec}$ under the same deformation. The speed bound is the lowest generalized eigenvalue of their pair through $G_{\rm acc}$.

In a finite or continuum model, a universal value of c_* would require the generalized spectrum of $(K_{\rm rec}, M_{\rm rec})$ to converge under refinement to the substrate access spectrum $G_{\rm acc}$, and to be independent of the admissible observer cut. That universality is not assumed here.

Compatibility without metric coincidence. The substrate framework requires only

$$\operatorname{spec}\big(M_{\rm rec}^{-1/2} K_{\rm rec} M_{\rm rec}^{-1/2}\big) \;\to\; \operatorname{spec}(G_{\rm acc}),$$

where $G_{\rm acc}$ is the substrate's intrinsic access-spectrum operator. It does not impose

$$K_{\rm rec} \;=\; M_{\rm rec}^{1/2}\, L_{\rm acc}\, M_{\rm rec}^{1/2}$$

as a definition. The metric-compatible form is a possible realization of the compatibility condition — one in which $G_{\rm acc}$ literally equals the access Laplacian $L_{\rm acc}$ — but the framework only requires that the generalized ratio of the two independently constructed operators stabilizes under refinement to the same access spectrum. Both $K_{\rm rec}$ and $M_{\rm rec}$ are then nontrivial substrate-built operators, and neither is defined from the other.

Extreme regimes. The same paired bandwidth formula gives precise framework-native placements for several limiting cases. In a black-hole-like region B, exterior recovery fails when the entropy susceptibility carried by $M^{\rm ext}_{{\rm rec},B}$ overwhelms the support stiffness carried by $K^{\rm ext}_{{\rm rec},B}$,

$$\lambda_{\min}\Big((M^{\rm ext}_{{\rm rec},B})^{-1/2} K^{\rm ext}_{{\rm rec},B} (M^{\rm ext}_{{\rm rec},B})^{-1/2}\Big) \;\to\; 0.$$

Then

$$c^2_{*,{\rm ext}}(B) \;\to\; 0,$$

so the horizon is an exterior record-bandwidth collapse rather than a primitive substrate wall. Interior information need not be destroyed in W; it becomes nonrecoverable to the exterior algebra because the reverse channel is dominated by record susceptibility at the boundary cut.

In quantum regimes, the defect covariance P_D and the record susceptibility $M_{\rm rec}$ dominate the observer readout. Large $M_{\rm rec}$-eigenvalue ratios relative to $K_{\rm rec}$ produce probabilistic, noisy, or nonseparable records; small ratios give stiff, low-noise records and therefore a classical-looking stable history. The bandwidth law therefore places the quantum/classical transition in the generalized spectrum of $(K_{\rm rec}, M_{\rm rec})$.

In cosmological regimes, the relevant region is a large accessible domain Ω. The global disturbance baseline contributes to the observer-effective vacuum offset, while the large-scale bandwidth is

$$c^2_{*,\Omega} \;=\; \inf_v \frac{\langle v, K_{{\rm rec},\Omega} v\rangle}{\langle v, M_{{\rm rec},\Omega} v\rangle}.$$

A cosmological-constant-like term is then not a localized matter disturbance but the large-region baseline part of the paired disturbance-action dictionary. Its effect on large-scale causal appearance is mediated by the same pair of quantities: global stiffness through $K_{\rm rec}$ and global record susceptibility through $M_{\rm rec}$.

Finite generalized-eigenvalue benchmark. The generalized-eigenvalue statement can be tested in a deliberately minimal finite model without assigning any physical value to c_*. Let the admissible support-deformation space be $V_R \cong \mathbb{R}^6$. Choose two positive-definite matrices realizing the structural decompositions

$$\begin{aligned} K_{\rm rec} &= K_0 + \kappa_{\rm acc}K_{\rm acc} + \kappa_{\rm str}K_{\rm str} + \kappa_{\rm ns}K_{\rm ns} + \kappa_{\rm sec}K_{\rm sec}, \\ M_{\rm rec} &= M_0 + \eta_{\rm acc}M_{\rm acc} + \eta_{\rm str}M_{\rm str} + \eta_{\rm ns}M_{\rm ns} + \eta_{\rm sec}M_{\rm sec}, \end{aligned}$$

with each ingredient SPD on V_R and the coefficients chosen so that $K_{\rm rec}, M_{\rm rec} > 0$. For a direct record-stability simulation, a deformation mode v updated at inclusion rate r is declared stable exactly when

$$r^2\langle v, M_{\rm rec}v\rangle \;\le\; \langle v, K_{\rm rec}v\rangle.$$

The eigenvalue prediction is not separately fitted; it is the Rayleigh-quotient threshold

$$c_*^2 \;=\; \lambda_{\min}\big(M_{\rm rec}^{-1/2}K_{\rm rec}M_{\rm rec}^{-1/2}\big) \;=\; \inf_{v\neq 0}\frac{\langle v, K_{\rm rec}v\rangle}{\langle v, M_{\rm rec}v\rangle}.$$

In a representative run using the structural decomposition with simple admissible choices for each ingredient and unit normalization weights, the generalized eigenvalue calculation reproduces the same critical threshold to which Monte Carlo random-direction sampling converges, and the critical-direction test makes the transition explicit: rates below the threshold leave the inequality positive, rates equal to the threshold give equality on the critical eigenvector, and rates above the threshold violate the inequality. This is the logical necessity test of the paired generalized eigenvalue construction: when the same $(K_{\rm rec}, M_{\rm rec})$ define direct record stability, the first unstable mode is exactly the generalized eigenmode, and the record-bandwidth threshold is the corresponding eigenvalue bound on $G_{\rm acc}$.

Stability envelope and null record modes. The record-bandwidth bound can be phrased geometrically without assigning a numerical value to c_*. For a region R, define the local record-stability envelope

$$\mathcal{C}_R \;:=\; \big\{(r,v) : r^2\langle v, M_{\rm rec}v\rangle \le \langle v, K_{\rm rec}v\rangle\big\}.$$

Its boundary,

$$r^2\langle v, M_{\rm rec}v\rangle \;=\; \langle v, K_{\rm rec}v\rangle,$$

is the null recoverability boundary of the observer cut. The critical modes that first saturate this boundary satisfy the generalized eigenvalue problem

$$K_{\rm rec}v_* \;=\; c_*^2\, M_{\rm rec}v_*.$$

Thus the generalized eigenvector is not, by itself, a photon or a physical light ray. It is the first saturating algebraic deformation mode of the paired record-stability problem. A photon-like interpretation would require an additional identification: the saturating eigenspace must lie in the accessible holonomy/gauge sector, carry the appropriate transverse degrees of freedom, and obey the Maxwell-type stationarity condition of the low-disturbance readout. The framework therefore treats critical eigenvectors as *null record modes*; identifying a null record mode with light is a reduction target, not an assumption.

The same language clarifies the failure modes. If $c_* = \infty$, there is no finite recoverability cone and the observer layer becomes Galilean-like rather than Lorentzian. If $c_* = 0$ in an ordinary region, distinct stable records cannot propagate and extended dynamics fails. If $c_*(R, v)$ depends on direction, the local geometry is anisotropic, closer to a Finsler or medium-like record geometry than to Lorentz spacetime. If different admissible observers disagree about the boundary $r^2\langle v, M_{\rm rec} v\rangle = \langle v, K_{\rm rec} v\rangle$, then there is no observer-independent causal structure. Consequently the eigenvalue bound supplies the speed scale, but Lorentz kinematics requires the additional invariant-cone hypotheses stated in the next subsection.

For black-hole-like cuts, the same envelope gives a precise threshold formulation. A horizon is approached when the exterior envelope collapses in the interior-to-exterior directions,

$$\lambda_{\min}\big((M^{\rm ext}_{{\rm rec},\partial B})^{-1/2} K^{\rm ext}_{{\rm rec},\partial B} (M^{\rm ext}_{{\rm rec},\partial B})^{-1/2}\big) \;\to\; 0,$$

or falls below the recovery threshold of the exterior observer. This resembles an entropy-area constraint only after one proves that the boundary stiffness scales with area and that the susceptibility readout coincides with physical entropy. Recovering the Bekenstein–Hawking [33, 34] coefficient is therefore a separate reduction target, not a result of the present finite diagnostic.

Light as the saturating record sector. The electromagnetic or gauge/light sector is identified with the finite speed only when its records saturate the recoverability bound. More precisely, if the massless gauge-record sector is the minimal-disturbance stable sector for which

$$d_\omega(R_\lambda, R_{\lambda^+}) \;=\; c_{\mathcal{O}}\, \Delta\tau_\lambda$$

on admissible transitions, then

$$c_{\rm light} \;=\; c_{\mathcal{O}}.$$

Thus the framework does not say that light measures how fast the substrate moves, and it does not require $c_{\rm light} = v_{\rm sub}$. It says that light is the fastest stable way an observer can receive decohered, monotonically recoverable records through the substrate–observer junction. The electromagnetic sector inherits the paired bookkeeping with a matching coupling renormalization:

$$\boxed{K^{\rm em}_{\rm rec} \;=\; g_{\rm eff}^{-2}\, K_{\rm rec}, \qquad M^{\rm em}_{\rm rec} \;=\; g_{\rm eff}^{-2}\, M_{\rm rec}.}$$

The matched pair ensures that the electromagnetic record cone

$$c^2_{\rm em} \;=\; \lambda_{\min}\big((M^{\rm em}_{\rm rec})^{-1/2} K^{\rm em}_{\rm rec} (M^{\rm em}_{\rm rec})^{-1/2}\big) \;=\; \lambda_{\min}\big(M^{-1/2}_{\rm rec} K_{\rm rec} M^{-1/2}_{\rm rec}\big) \;=\; c^2_*$$

coincides with the universal record cone: the $g_{\rm eff}^{-2}$ factor cancels exactly between numerator and denominator, so the electromagnetic coupling $g_{\rm eff}$ controls the Maxwell amplitude scale without altering the universal record cone. This is the speed that enters the Lorentz recovery below. We write $c_* := c_{\mathcal{O}}$ throughout.

7.3 Recovered Lorentz kinematics from invariant modular-access cones

Lorentz kinematics is the hardest sector. The preceding section gives a finite observer-junction speed c_* as a maximum stable record-access rate determined by the lowest generalized eigenvalue of $G_{\rm acc}$. A finite speed alone, however, is still not enough to force Lorentz transformations. The stronger substrate–observer recovery requires an invariant access cone: all admissible observers must agree on which record events can be joined by finite record recoverability.

The substrate supplies an algebraic distance, not a coordinate distance. For regions or record supports R, S, write $d_\omega(R, S)$ for the state-dependent algebraic distance defined by conditional-expectation attenuation, modular distinguishability, or shell depth in the algebraic coarsening category. The observer supplies stable record increments and cut updates constrained by the Finite Record Recoverability Principle. The junction principle is invariant accessibility:

$$d_\omega(R_\lambda, R_\mu) \;\le\; c_*\, |\tau_\lambda - \tau_\mu|$$

must have the same truth value for every observer using the same modular state class and stable cut category.

When the accessible record histories admit a two-dimensional normal form spanned by record duration and one support-separation variable, the invariant of this access cone is

$$ds^2_{\mathrm{acc}} \;=\; c_*^2\, d\tau^2 - d_\omega^2.$$

Transformations preserving the access cone preserve ds^2_{acc}, and the usual Lorentz factor follows in an observer chart:

$$d\tau_{\mathrm{prop}} \;=\; dt\sqrt{1 - v^2/c_*^2}, \qquad \gamma \;=\; (1 - v^2/c_*^2)^{-1/2}.$$

A stable excitation with support stiffness m then has effective action

$$S_{\mathrm{obs}} \;=\; -mc_*^2 \int d\tau_{\mathrm{prop}},$$

which gives

$$p \;=\; \gamma m v, \qquad E \;=\; \gamma m c_*^2, \qquad E^2 - p^2 c_*^2 \;=\; m^2 c_*^4.$$

At rest, $E_0 = mc_*^2$.

This is a recovery only under the invariant modular-access-cone hypothesis. It is stronger than a bandwidth compatibility statement, but it still marks the open technical point honestly: on a purely tracial Type II_1 substrate the modular flow is trivial, so nontrivial Lorentzian access geometry requires either a Type III extension or an observer cut whose accessible algebra carries nontrivial modular data. The law is recovered at the junction; it is not proved from the Type II_1 substrate alone.

7.4 Wavefunction and collapse from accessible-algebra restriction

The observer statement also hosts the standard mathematical structure of a wavefunction and measurement collapse. The interpretive restriction is essential: the wavefunction is not the state of the entire substrate, and collapse is not a change of the substrate state. Both are observer-effective objects associated with an accessible algebra and a recorded outcome.

Let $\mathcal{O} = (\Lambda, \preceq, \{A_\lambda\}, \{M_\lambda\})$ be the observer. At label λ, the observer has access only to $A_\lambda \subseteq W$. The observer-effective state is the restriction

$$\omega_\lambda \;=\; \omega|_{A_\lambda}.$$

Represented on a Hilbert space, it may be written as a density operator ρ_λ. If the restricted state is pure, one may write $\rho_\lambda = |\psi_\lambda\rangle\langle\psi_\lambda|$. The wavefunction is therefore read as the observer-effective state on the accessible algebra. It is not required to be a fundamental substrate object.

Let a record-compatible measurement inside A_λ be represented by projectors $\{P_k\} \subset A_\lambda$. The observer assigns Born probabilities

$$p(k) \;=\; \omega_\lambda(P_k) \;=\; \mathrm{Tr}(P_k \rho_\lambda).$$

If outcome k is written into the later record algebra $M_{\lambda'}$, the observer-effective state updates by Lüders conditionalization,

$$\rho_{\lambda'} \;=\; \frac{P_k \rho_\lambda P_k}{\mathrm{Tr}(P_k \rho_\lambda)}.$$

The substrate state ω on W is not modified by this update. What changes is the observer's conditional state on the accessible algebra after the record has been written. Collapse is record-conditioned access, not substrate-level dynamics.

7.5 Recovered Maxwell-type equations from a paired holonomy readout

The Maxwell sector is recovered by applying the same junction rule to the substrate's internal connection sector through the paired $(K_{\rm rec}, M_{\rm rec})$ bookkeeping. The substrate supplies a compact internal automorphism sector with Abelian edge transport in the weak-field sector. The observer supplies accessible gauge records and conserved source records. The junction principle is stationary low-disturbance holonomy response, understood as the Abelian connection readout of the paired master regional disturbance.

Let A be the observer-accessible Abelian connection 1-cochain and let

$$F = dA$$

be its curvature. The identity

$$dF = d^2 A = 0$$

is substrate-side algebra: it follows from nilpotency of the coboundary and requires no source equation. The sourced equation is recovered only after an observer cut supplies a conserved accessible current J. Conservation is not a separate dynamical law here; it is the compatibility condition that the current record can be written through the same access cut without creating or destroying charge inside the inaccessible complement:

$$\delta J = 0.$$

Define the holonomy-disturbance action on the accessible connection sector through the matched pair $(K^{\rm em}_{\rm rec}, M^{\rm em}_{\rm rec}) = (g_{\rm eff}^{-2} K_{\rm rec}, g_{\rm eff}^{-2} M_{\rm rec})$ by

$$\mathcal{A}_{\rm em}[A; J] = \frac{1}{2g_{\rm eff}^2} \langle dA, dA \rangle_\omega - \langle J, A \rangle_\omega,$$

where the inner product is the observer-accessible quadratic form induced by the cut, the state, and the substrate's connection data. This action is not an independent primitive sector; it is the Abelian holonomy-sector quadratic expansion of $\mathcal{K}_R$ under accessible connection variations, with the matched paired bookkeeping in the electromagnetic sector. Stationarity with respect to accessible gauge variations $A \mapsto A + \eta$, modulo pure-gauge directions, gives

$$\delta dA = g_{\rm eff}^2 \, J.$$

Together with $F = dA$, this yields the Maxwell-type system

$$dF = 0, \qquad \delta F = g_{\rm eff}^2 \, J.$$

Thus the field equations are not obtained by defining J from F. The current is an observer-accessible conserved record, while F is the stationary low-disturbance holonomy response to that record. The coupling $g_{\rm eff}$ controls the amplitude through the matched pair, but does not alter the universal record cone $c_*^2 = \lambda_{\rm min}(G_{\rm acc})$, because the $g_{\rm eff}^{-2}$ factor cancels between the matched $K^{\rm em}_{\rm rec}$ and $M^{\rm em}_{\rm rec}$.

Finite $U(1)$ stationarity benchmark. The validation companion includes a finite sourced $U(1)$ solve in which the accessible connection is varied while the same paired master disturbance supplies the quadratic holonomy cost. The important point of that computation is not that a lattice script proves Maxwell theory. It is that relaxing the accessible connection lowers the single paired disturbance functional and satisfies the discrete stationarity equation in the presence of a conserved current. This is the finite counterpart of the claim made here: the Abelian gauge law is an accessible-connection stationarity readout of $\mathcal{K}_R$, not a separately inserted strain sector.

Gauge invariance is also a junction property. Under $A \mapsto A + d\chi$,

$$F \;\mapsto\; dA + d^2\chi \;=\; F,$$

and the source term changes by

$$\langle J, d\chi\rangle_\omega \;=\; \langle \delta J, \chi\rangle_\omega \;=\; 0$$

for conserved accessible current. Therefore gauge invariance and charge conservation are two sides of the same access-compatible stationarity condition.

Non-Abelian selection. The same recovery pattern also explains when a non-Abelian sector is needed. Let edge transports take values in a compact group G, with

$$U_{ij} \in G, \qquad U_{ji} = U_{ij}^{-1}.$$

For a loop $\gamma = (i_0, i_1, \ldots, i_n, i_0)$, define

$$H_\gamma = U_{i_0 i_1} U_{i_1 i_2} \cdots U_{i_n i_0}.$$

If all admissible coarsening transitions commute, an Abelian phase sector is sufficient. If there are admissible low-disturbance coarsenings with

$$[T_{RS}, T_{ST}] \neq 0,$$

then Abelian holonomy cannot represent the order-dependence of the regional join. The low-disturbance stationary connection must then live in a noncommutative compact sector, and closed-loop mismatch is measured by the conjugacy-invariant Wilson defect

$$W_\gamma \;=\; \operatorname{Re}\operatorname{Tr}(I - H_\gamma).$$

This does not derive the Standard Model. It gives a selection rule: non-Abelian holonomy is recovered when algebraic coarsening itself contains noncommuting transport data that cannot be encoded by $U(1)$ phases.

7.6 Recovered weak-field gravity from stationary accessible connection response

The weak-gravity sector is recovered from the same substrate–observer pattern. The substrate supplies localized regional disturbance and the paired connection-derived geometry $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$. The observer supplies an accessible scalar or metric-like connection potential. The junction principle is stationary accessible connection response to paired disturbance.

Let X be a persistent excitation. Its source is not independently declared; it is the accessible density associated with the local disturbance increment,

$$T[X] \;=\; \Pi_0\big(\delta\mathcal{K}_R(X)\, p_X\big),$$

where p_X is the support profile and Π_0 removes the null mode of the accessible connection operator. Let Δ_∇ be the positive connection-stiffness operator induced by K_{rec} on the accessible cut. The observer-accessible potential ϕ is recovered by stationarity of

$$\mathcal{A}_{\mathrm{grav}}[\phi; X] \;=\; \tfrac{1}{2}\langle \phi, \Delta_\nabla \phi\rangle_\omega - \kappa\langle T[X], \phi\rangle_\omega.$$

For all admissible variations η in the gauge-fixed accessible sector,

$$\delta\mathcal{A}_{\mathrm{grav}}[\phi; X](\eta) \;=\; \langle \eta, \Delta_\nabla\phi - \kappa T[X]\rangle_\omega.$$

Stationarity gives the recovered weak-field equation

$$\Delta_\nabla \phi \;=\; \kappa\, T[X].$$

This has the same logical form as the recovered thermodynamic balance: a substrate quantity, here $\delta\mathcal{K}_R(X)$, is paired with an observer-access quantity, here ϕ, and the effective equation is the stationarity condition at their junction. The equation is not a postulated Poisson law; it is the low-disturbance accessible connection response to a paired regional disturbance source.

Mass-independent test acceleration is recovered as a weak-limit consequence of the shared-generator principle. Let Y be a test excitation with support-stiffness scalar $m_Y > 0$ carried by K_{rec}. Its inertial readout is the support Hessian of $\mathcal{K}_R$, while its gravity-like coupling is the connection variation of the same $\mathcal{K}_R$. In the weak, isotropic, low-disturbance limit both readouts reduce to the same trace-normalized coefficient. The force read by the observer is therefore

$$F_{\mathrm{grav}}(Y) \;=\; m_Y\, g_\nabla, \qquad g_\nabla = -\nabla_{\mathrm{acc}}\phi,$$

while the inertial readout is

$$m_Y\, a_Y \;=\; F_{\mathrm{grav}}(Y).$$

Therefore

$$a_Y \;=\; g_\nabla,$$

independent of the test support's own disturbance magnitude. If Y is composite, Corollary 8.2 of Section 8 implies that binding, kinetic, electromagnetic, and rest-like internal contributions all enter through the same leading paired regional disturbance geometry; the cancellation is therefore composition-insensitive in the weak-limit observer readout.

This recovery is still weak-field and observer-effective. It is not a derivation of Einstein's equation, diffeomorphism invariance, or Lorentzian geodesic motion. Those require the Type III/modular extension and the invariant access-cone construction. What is recovered here is the Newtonian weak-field structure: paired regional disturbance sources an accessible connection, and test support acceleration is independent of the test body's own disturbance coefficient because the same paired master disturbance defines both inertial and gravity-like readouts in the weak limit.

8 Algebraic necessity program: paired disturbance, access, shell growth, non-Abelian selection, and observer cuts

The preceding sections define the observer-effective readouts on top of the algebraic substrate. The necessity program records the stronger claim that the framework must pursue: the familiar equations should not merely be compatible with engineered readouts, but should arise as stability or stationarity conditions of the substrate itself. This section recasts four remaining choices as substrate-facing derivation targets. The claims below remain programmatic in part, but they remove coordinate assumptions from the substrate core and are formulated so that future numerics and operator-algebraic models can test them without appealing to an ambient coordinate system.

8.1 Conditional-expectation defects and the Type II_1 isotropic quadratic disturbance theorem

The shared-generator use of $\mathcal{K}_R$ should not remain a bare physical postulate. A rigorous foundation result of the framework is that, in a real finite von Neumann factor, the proposed quadratic paired disturbance geometry is not merely a matrix-proxy choice once an isotropic local defect sector is assumed. It is the local normal form of a positive, trace-covariant, isotropic functional measuring the failure of regional conditional expectations to compose over a join, and it controls both the stiffness $K_{\rm rec}$ and the susceptibility $M_{\rm rec}$ through their common canonical trace.

Let M be a Type II_1 factor with faithful normal tracial state τ. Let $A, B \subset M$ be von Neumann subalgebras and let

$$C := A \vee B$$

be the von Neumann algebra they generate. Since M is finite, each von Neumann subalgebra $N \subset M$ admits a unique τ-preserving conditional expectation

$$E_N : M \to N.$$

The trace defines the noncommutative Hilbert space

$$L^2(M, \tau), \qquad \|x\|_{2,\tau}^2 := \tau(x^*x),$$

and each E_N extends to the orthogonal projection

$$P_N : L^2(M, \tau) \to L^2(N, \tau).$$

Define the regional join defect on the joined support by

$$D_{A,B} \;:=\; (P_C - P_A P_B)|_{L^2(C,\tau)}.$$

Equivalently, for $x \in L^2(C, \tau)$,

$$D_{A,B}x \;=\; x - E_A E_B x.$$

This defect measures the part of a joined regional perturbation that is lost or distorted when the joined description is compressed sequentially through the two regional expectations.

Theorem 8.1 (Type II_1 isotropic quadratic disturbance normal form)**.** *Let M be a Type II_1 factor with trace τ, and let $A, B \subset M$ be von Neumann subalgebras with $C = A \vee B$. Then*

$$\sigma_{A,B}(x) \;:=\; \|D_{A,B}x\|_{2,\tau}^2, \qquad x \in L^2(C, \tau),$$

is a positive, trace-intrinsic, internally unitarily covariant quadratic disturbance functional on local support perturbations. Moreover, if a local disturbance functional F of the defect variable satisfies

1. *$F(0) = 0$;*
2. *$F(y) \geq 0$ near $y = 0$;*
3. *F is twice Fréchet differentiable at 0;*
4. *F is invariant under trace-preserving internal unitary conjugacies; and*
5. *the local defect sector is isotropic, so no internal defect direction is assigned a preferred stiffness,*

then its leading nonzero term has the form

$$F(D_{A,B}x) \;=\; c\,\|D_{A,B}x\|_{2,\tau}^2 + o(\|D_{A,B}x\|_{2,\tau}^2), \qquad c \geq 0.$$

If the disturbance readout is nondegenerate on the defect sector, then $c > 0$.

Proof. Because M is a Type II_1 factor, τ is finite, faithful, normal, and tracial. Hence

$$\langle x, y\rangle_{2,\tau} := \tau(y^*x)$$

defines a Hilbert-space inner product on $L^2(M,\tau)$. For every von Neumann subalgebra $N \subset M$, the τ-preserving conditional expectation E_N is L^2-contractive and extends to the orthogonal projection P_N onto $L^2(N,\tau)$. Therefore P_A, P_B, P_C are orthogonal projections on $L^2(M,\tau)$, and $D_{A,B}$ is a bounded local defect operator on $L^2(C,\tau)$. It follows immediately that

$$\sigma_{A,B}(x) = \langle D_{A,B}x, D_{A,B}x\rangle_{2,\tau} \geq 0,$$

with equality precisely when $D_{A,B}x = 0$.

Let $u \in M$ be a unitary implementing an internal relabeling of the regional data:

$$A \mapsto uAu^*, \quad B \mapsto uBu^*, \quad C \mapsto uCu^*.$$

Since τ is tracial,

$$\|uxu^*\|_{2,\tau}^2 = \tau(ux^*xu^*) = \tau(x^*x) = \|x\|_{2,\tau}^2.$$

The expectations transform covariantly,

$$E_{uNu^*}(uxu^*) = uE_N(x)u^*,$$

so

$$D_{uAu^*,uBu^*}(uxu^*) = uD_{A,B}(x)u^*.$$

Thus

$$\sigma_{uAu^*,uBu^*}(uxu^*) = \sigma_{A,B}(x),$$

which proves internal unitary covariance.

Now let F be any positive twice differentiable local disturbance functional of a defect variable y with $F(0) = 0$. Since 0 is a local minimum, the first variation vanishes: $DF(0) = 0$. Taylor expansion gives

$$F(y) = \tfrac{1}{2}D^2F(0)[y,y] + o(\|y\|_{2,\tau}^2).$$

The Hessian is a positive semidefinite quadratic form. Internal unitary covariance and local isotropy rule out preferred directions in the defect sector; the only admissible leading quadratic form is therefore a scalar multiple of the L^2 inner product. Hence

$$D^2F(0)[y,y] = 2c\|y\|_{2,\tau}^2$$

for some $c \geq 0$. Setting $y = D_{A,B}x$ proves the claim. □

The theorem provides the substrate-level normal form used by the physical reading of the paper. The Type III_1 case relevant to local relativistic quantum field theory remains a modular version of this result: one must replace the trace geometry by the faithful-state GNS/modular geometry and control the existence or replacement of ω-preserving expectations. But the finite factor theorem already removes the main matrix-proxy objection: quadratic paired regional disturbance is the local normal form of conditional-expectation defect under standard stability and covariance hypotheses, and the canonical trace anchors both K_{rec} and M_{rec} to the same Hilbert–Schmidt geometry.

Corollary 8.2 (Shared disturbance geometry and composition independence)**.** *Assume the Type* II_1 *hypotheses above and let a persistent composite support be described by defect variables* $D_{R,S}^{X_1,\ldots,X_k}$ *for all relevant regional joins. If its total substrate disturbance is the sum of local defect contributions,*

$$\Sigma_A(X_1,\ldots,X_k) \;=\; \sum_{R,S\subset A} F\left(D_{R,S}^{X_1,\ldots,X_k}\right),$$

with the same positive, covariant local functional F satisfying Theorem 8.1, then every contribution to the composite support enters through the same leading paired disturbance coefficient c:

$$\Sigma_A = c \sum_{R,S\subset A} \|D_{R,S}^{X_1,\dots,X_k}\|_{2,\tau}^2 + o\left(\sum_{R,S} \|D_{R,S}^{X_1,\dots,X_k}\|_{2,\tau}^2\right).$$

Therefore the inertial Hessian carried by K_{rec} and the gravity-like connection variation of the composite inherit one common trace-normalized disturbance geometry, and the record susceptibility carried by M_{rec} matches through the same canonical trace. Internal composition can change the shape and magnitude of the defect, but it cannot introduce a second canonical coupling for binding, kinetic, electromagnetic, or rest-like contributions.

Proof. Apply Theorem 8.1 to each local defect contribution. The same trace τ and the same local normal form determine each leading quadratic term. Summing defects changes the total quadratic form but not the source of its overall coefficient. The inertial stiffness and gravity-like connection response are variational readouts of this same total disturbance with respect to support and connection parameters, so they inherit the same local paired quadratic geometry. Distinct composition-dependent couplings would require distinct canonical quadratic geometries on the same defect sector, contradicting the single normal form under the stated covariance and isotropy hypotheses. □

This corollary is the paper's substrate-side version of composition independence. It says that if all physical contributions enter as defects of one persistent regional support, then all such contributions participate in the same paired disturbance geometry. The result does not yet produce spacetime free fall, but it prevents the substrate from assigning one gravity-like coefficient to rest-like persistence and another to internal binding contribution.

Tracial limitation. The same Type II_1 trace that makes Theorem 8.1 available also makes modular flow trivial:

$$\sigma_t^\tau = \mathrm{id}.$$

Hence the Type II_1 theorem cannot by itself generate Connes–Rovelli thermal time, observer arrow, or Lorentzian kinematics. The allocation is explicit: Type II_1 supplies a clean substrate normal form and the shared-generator limit; nontrivial observer time and thermodynamics require state-dependent modular structure on the observer-access side, or a future Type III_1 extension of the theorem.

8.2 Spectral covariance of defect modes from observer coarse-graining

The previous theorem fixes the local quadratic geometry of conditional-expectation defect, but it does not yet explain why a paired trace-quadratic disturbance should have the kind of continuum asymptotics that can produce an Einstein–Hilbert term. The missing bridge is spectral. If the generalized record operator $G_{\text{acc}} = M_{\text{rec}}^{-1/2} K_{\text{rec}} M_{\text{rec}}^{-1/2}$ has a Laplace-type continuum limit, and if the observer-access defect covariance is controlled only by the spectrum of G_{acc}, then the master disturbance becomes a spectral weighted trace. Heat-kernel methods can then generate a local expansion in volume, scalar curvature, and higher-curvature invariants.

This subsection records the precise justification for the spectral assumption. It is not an additional physical law. It is the least-biased covariance assignment available to an observer whose access algebra cannot distinguish microscopic defect labels beyond their disturbance cost under the paired geometry.

Exact finite spectral rewriting. Let $\mathcal{H}_D(R)$ be the finite defect Hilbert space associated with the regional defect $\mathfrak{D}_R$. Let

$$K := K_{\mathrm{rec}}(\nabla, g, R) \geq 0$$

be the stiffness operator on that defect space and $M := M_{\mathrm{rec}}(\nabla, g, R) > 0$ the matching susceptibility. The generalized record operator is

$$G_{\mathrm{acc}} = M^{-1/2} K M^{-1/2}.$$

If $G_{\mathrm{acc}}\phi_\alpha = \lambda_\alpha \phi_\alpha$, and

$$\mathfrak{D}_R = \sum_\alpha d_\alpha \, M^{1/2} \phi_\alpha,$$

then the finite disturbance is exactly

$$\mathcal{K}_R = \langle \mathfrak{D}_R, K \mathfrak{D}_R \rangle = \sum_\alpha \lambda_\alpha \, |d_\alpha|^2 \cdot \langle \phi_\alpha, \phi_\alpha \rangle.$$

Equivalently, if P_D denotes the defect covariance or density on $\mathcal{H}_D(R)$, then

$$\boxed{\mathcal{K}_R \; = \; \mathrm{Tr}(K \, P_D)}$$

and, in the G_{acc}-normalized basis,

$$\mathcal{K}_R \; = \; \mathrm{Tr}\big(G_{\mathrm{acc}} \, M^{1/2} P_D M^{1/2}\big).$$

Thus the master disturbance is always a paired spectral energy once the defect space and the operators $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ are fixed. What is not automatic is that this weighted trace becomes a universal spectral action rather than a mode-by-mode memory of microscopic defect preparation.

Spectral covariance as the universal regime. A universal spectral action follows if the defect covariance is approximately a function of the generalized record operator,

$$P_D \approx f(G_{\mathrm{acc}}/\Lambda^2).$$

Then

$$\mathcal{K}_R = \mathrm{Tr}\big(K \, f(G_{\mathrm{acc}}/\Lambda^2)\big) = \Lambda^2 \, \mathrm{Tr} \, F(G_{\mathrm{acc}}/\Lambda^2), \qquad F(x) := x f(x),$$

which is a spectral-action-type expression. The question is therefore not whether $\mathcal{K}_R$ can be diagonalized. It can. The question is why observer-access covariance should commute with G_{acc}, so that only spectral shells matter.

Coarse-graining justification. The observer does not access the substrate's full microscopic labels. The observer has stable access only to record-preserving quantities. If G_{acc} is the generator of recoverability cost (the lowest eigenvalue of G_{acc} being c_*^2), then two defect modes in the same spectral shell of G_{acc} have the same observable disturbance cost. An observer whose algebra contains no additional stable labels cannot assign a preferred basis inside such a shell. Therefore the covariance must be invariant under all unitaries that preserve G_{acc}. This is the algebraic meaning of coarse-graining over unobserved defect labels.

Theorem 8.3 (Spectral covariance from observer coarse-graining)**.** *Let G_{acc} be the positive self-adjoint generalized record operator on a finite regional defect Hilbert space $\mathcal{H}_D$, and let $P_D \geq 0$ be the observer-access defect covariance. Suppose:*

1. *the observer fixes the total accessible defect weight $\mathrm{Tr}\, P_D = N_D$;*

2. the observer fixes the mean disturbance $\mathrm{Tr}(G_{\mathrm{acc}}\,P_D) = \mathcal{E}_D$*;*

3. the observer has no stable labels inside a spectral shell of G_{acc}*, so* $UG_{\mathrm{acc}}U^* = G_{\mathrm{acc}} \implies UP_DU^* = P_D$ *for every unitary* U *on* $\mathcal{H}_D$*.*

Then P_D *commutes with* G_{acc} *and is block-spectral:*

$$P_D = \sum_\lambda p_\lambda \Pi_\lambda,$$

where Π_λ *is the spectral projection of* G_{acc}*. In the nondegenerate or smooth-shell limit this is equivalently*

$$P_D = f(G_{\mathrm{acc}}/\Lambda^2)$$

for some spectral filter f *and access scale* Λ*. If, in addition,* P_D *maximizes von Neumann entropy subject to the two constraints above, then*

$$\boxed{P_D = Z^{-1}\exp[-\beta G_{\mathrm{acc}}/\Lambda^2].}$$

Proof. Let $G_{\mathrm{acc}} = \sum_\lambda \lambda \Pi_\lambda$ be the spectral decomposition. If $UG_{\mathrm{acc}}U^* = G_{\mathrm{acc}}$, then U is arbitrary inside each eigenspace $\Pi_\lambda \mathcal{H}_D$ and cannot mix eigenspaces with distinct eigenvalues. The invariance condition $UP_DU^* = P_D$ for all such U implies, by Schur's lemma on each eigenspace, that P_D is scalar on every eigenspace and has no off-diagonal blocks between distinct eigenspaces. Hence $P_D = \sum_\lambda p_\lambda \Pi_\lambda$, so $[P_D, G_{\mathrm{acc}}] = 0$. If the spectrum is nondegenerate, set $f(\lambda/\Lambda^2) = p_\lambda$. If the eigenspaces are degenerate but the shell weights vary smoothly with λ, the same relation holds in the coarse spectral calculus.

For the maximum-entropy statement, maximize $S(P_D) = -\,\mathrm{Tr}(P_D \log P_D)$ subject to $\mathrm{Tr}\,P_D = N_D$ and $\mathrm{Tr}(G_{\mathrm{acc}}P_D) = \mathcal{E}_D$. Variation gives $-\log P_D - I - \alpha I - \beta G_{\mathrm{acc}} = 0$, so $P_D = Ce^{-\beta G_{\mathrm{acc}}}$, with $C = Z^{-1}$ fixed by $\mathrm{Tr}\,P_D = N_D$. Restoring the observer access scale gives the displayed form. □

Maximum entropy and modular equilibrium. The theorem gives two compatible readings. In the coarse-graining reading, $P_D = f(G_{\mathrm{acc}}/\Lambda^2)$ follows from ignorance of microscopic labels inside spectral shells. In the maximum-entropy reading, the exponential filter follows because the observer fixes only total defect weight and average disturbance. If the observer representation is nontracial, the same structure can be read modularly. Let

$$H_{\mathrm{mod}} = -\log \rho_{\mathcal{O}}$$

be the modular Hamiltonian of the observer-access state. In local equilibrium of the defect sector one expects a KMS/Gibbs form $P_D \propto e^{-\beta H_{\mathrm{mod}}}$. If the modular generator is controlled in the defect sector by the generalized record operator, $H_{\mathrm{mod}} \approx G_{\mathrm{acc}}/\Lambda^2$, then

$$P_D \approx Z^{-1} e^{-\beta G_{\mathrm{acc}}/\Lambda^2}.$$

Laplace-type continuum bridge. If, under refinement of the regional substrate,

$$G_{\mathrm{acc}}(\nabla_n, g_n, R) \;\longrightarrow\; -\nabla_g^2 + E$$

as a Laplace-type operator on an observer-effective continuum geometry, then the spectral weighted trace becomes amenable to heat-kernel asymptotics. In schematic form,

$$\mathrm{Tr}\,F(G_{\mathrm{acc}}/\Lambda^2) \sim c_0\Lambda^d \int_M \sqrt{|g|}\,d^dx + c_1\Lambda^{d-2}\int_M R\sqrt{|g|}\,d^dx + c_2\Lambda^{d-4}\int_M \mathcal{R}_2\sqrt{|g|}\,d^dx + \cdots,$$

where $\mathcal{R}_2$ denotes quadratic curvature invariants and bundle-curvature terms. Thus the regional disturbance has the possible low-curvature form

$$\mathcal{K}_R^{\mathrm{geom}} \sim \int_M (a_0 + a_1 R + a_2 R^2 + a_3 R_{\mu\nu} R^{\mu\nu} + \cdots) \sqrt{|g|}\, d^d x.$$

In the long-distance observer regime, if $a_1 \neq 0$ and higher-curvature terms are suppressed, this becomes the Einstein–Hilbert reduction target

$$\mathcal{K}_R^{\mathrm{geom}} \approx \frac{1}{16\pi G_{\mathrm{eff}}} \int_M (R - 2\Lambda_{\mathrm{eff}}) \sqrt{|g|}\, d^d x.$$

The present theorem does not prove this final limit. It reduces the problem to showing that the generalized record operator is Laplace-type, local, and observer-diffeomorphism invariant in the continuum limit, and that the scalar-curvature coefficient does not vanish.

Where the argument can fail. The spectral reduction fails if the observer has stable access to microscopic labels not generated by G_{acc}, because then P_D need not commute with G_{acc}. It also fails if the defect sector is far from maximum-entropy or modular equilibrium, if G_{acc} has no local Laplace-type continuum limit, if nonlocal tails survive refinement, or if the spectral filter makes $a_1 = 0$. These failure modes are useful: they make the reduction falsifiable inside the framework. A substrate that cannot satisfy them may still support finite disturbance bookkeeping, but it would not supply a route to Einstein–Hilbert dynamics.

Reduction status. The result of this subsection is therefore a conditional bridge, not a completed derivation of general relativity. It proves that observer coarse-graining and maximum-entropy assignment force the defect covariance to be spectral whenever the generalized record operator is the only stable label. It also shows why this matters: the master disturbance then becomes a spectral weighted trace, and spectral traces of Laplace-type operators are the known mathematical setting in which volume, scalar curvature, and higher-curvature terms appear. The remaining derivation problem is sharp:

$$\tau(\mathfrak{D}_R^* K_{\mathrm{rec}} \mathfrak{D}_R) \implies \operatorname{Tr} F(G_{\mathrm{acc}}/\Lambda^2) \implies \int_M (R - 2\Lambda) \sqrt{|g|}\, d^d x,$$

after a controlled observer-continuum limit. The first arrow is supplied by spectral covariance through G_{acc}; the second still requires a Laplace-type continuum theorem and a nonzero scalar-curvature coefficient.

8.3 Continuum Einstein-translation of the junction variation

The preceding spectral bridge gives the correct mathematical place where an Einstein–Hilbert term can appear, but it does not by itself execute the macroscopic variation. This subsection records the exact continuum translation required by the gravity claim. It is deliberately stated as a conditional theorem schema, because the conclusion is only as strong as the continuum hypotheses: locality of the generalized record operator limit, spectral covariance of the defect sector, emergence of an observer-effective Lorentzian metric, control of boundary terms, and a conserved defect-source tensor.

The purpose is to separate three statements that are easy to conflate:

1. *Architecture*: gravity-like response is the connection or metric variation of the paired master regional disturbance.
2. *Continuum dictionary*: the long-distance observer representation rewrites that disturbance as a local action of an effective metric and defect sources.

3. *Einstein execution*: variation of that local action with respect to the effective metric gives the Einstein tensor on the geometric side and stress-energy on the defect side.

Only the third item answers the question: what does the variation actually produce?

Macroscopic dictionary. Let a stable observer scaling limit of access labels produce an effective continuum region Ω with metric-like field $g_{\mu\nu}$, volume density $\sqrt{|g|}\,d^dx$, and admissible metric variations $\delta g^{\mu\nu}$ compactly supported in Ω or accompanied by the appropriate boundary term. The dictionary is

$$\begin{aligned}
\text{paired spectral trace} &\longrightarrow \text{geometric effective action},\\
\operatorname{Tr} F(G_{\mathrm{acc}}/\Lambda^2) &\longrightarrow \int_\Omega \big(a_0\Lambda^d + a_1\Lambda^{d-2}R[g] + a_2\Lambda^{d-4}\mathcal{I}_2[g] + \cdots\big)\sqrt{|g|}\,d^dx,\\
\text{localized defect sector} &\longrightarrow \text{observer-effective matter/source action},\\
\mathfrak{D}_R, P_D, \mathcal{L}_R &\longrightarrow S_{\mathrm{def}}[g,\Psi],\\
\text{junction stationarity} &\longrightarrow \delta_g S_{\mathrm{obs}} = 0.
\end{aligned}$$

Here Ψ denotes the continuum fields or collective defect variables that summarize persistent regional disturbance at the observer scale. The term $\mathcal{I}_2[g]$ denotes curvature-squared and higher derivative invariants.

The observer-effective action therefore has the form

$$S_{\mathrm{obs}}[g,\Psi] = S_{\mathrm{geo}}[g] + S_{\mathrm{def}}[g,\Psi] + S_{\mathrm{bdry}}[g],$$

with

$$S_{\mathrm{geo}}[g] = \int_\Omega \big(A_0 + A_1R[g] + A_2\mathcal{I}_2[g] + \cdots\big)\sqrt{|g|}\,d^dx.$$

The constants A_i are not fitted gravitational couplings at this stage. They are continuum coefficients determined, if the reduction succeeds, by the spectral filter, cutoff scale, trace normalization, and the spectrum of G_{acc}.

Metric variation of the geometric term. Assume first that the long-distance regime is dominated by the volume and scalar-curvature terms,

$$S_{\mathrm{geo}}[g] = \int_\Omega (A_0 + A_1R[g])\sqrt{|g|}\,d^dx + S_{\mathrm{bdry}}[g].$$

The boundary functional is the observer-effective analogue of the Gibbons–Hawking–York correction [40, 41]: it is included, or variations are restricted, so that no uncontrolled normal-derivative variation remains on $\partial\Omega$. The standard metric variations are

$$\delta\sqrt{|g|} = -\tfrac{1}{2}\sqrt{|g|}\,g_{\mu\nu}\delta g^{\mu\nu},$$

and, after the boundary contribution is cancelled or fixed,

$$\delta\Big(\int_\Omega R\sqrt{|g|}\,d^dx\Big) = \int_\Omega G_{\mu\nu}\,\delta g^{\mu\nu}\sqrt{|g|}\,d^dx,$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$. Therefore

$$\delta S_{\mathrm{geo}} = \int_\Omega \big(A_1G_{\mu\nu} - \tfrac{1}{2}A_0g_{\mu\nu}\big)\,\delta g^{\mu\nu}\sqrt{|g|}\,d^dx + \delta S_{\mathrm{higher}},$$

where $\delta S_{\mathrm{higher}}$ is the variation of the curvature-squared and higher-order terms. In the low-curvature regime where those terms are negligible, the geometric variation produces precisely the Einstein tensor plus a cosmological term.

Defect-source variation and stress-energy. The matter/source side is not introduced as a separate gravitational charge. It is the continuum readout of persistent regional paired defect structure. Define the observer-effective stress tensor by the metric variation of the defect action:

$$T^{\text{def}}_{\mu\nu} := -\frac{2}{\sqrt{|g|}} \frac{\delta S_{\text{def}}}{\delta g^{\mu\nu}}.$$

Equivalently,

$$\delta S_{\text{def}} = -\tfrac{1}{2} \int_\Omega T^{\text{def}}_{\mu\nu}\, \delta g^{\mu\nu} \sqrt{|g|}\, d^d x.$$

This is the continuum form of the framework's source statement: the source is the metric/access variation of the same paired regional disturbance whose support Hessian through K_{rec} gives inertia. Rest-like, binding, electromagnetic, kinetic, gauge, and internal contributions may change the defect action S_{def}, but they enter a single tensor $T^{\text{def}}_{\mu\nu}$ because they are continuum components of one persistent defect sector rather than independent gravity-like charges.

Junction stationarity. The macroscopic junction condition is

$$\delta_g S_{\text{obs}}[g, \Psi] = 0$$

for all admissible metric-like observer variations. Combining the previous two variations gives

$$\int_\Omega \left(A_1 G_{\mu\nu} - \tfrac{1}{2} A_0 g_{\mu\nu} - \tfrac{1}{2} T^{\text{def}}_{\mu\nu} + H^{\text{higher}}_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{|g|}\, d^d x = 0,$$

where $H^{\text{higher}}_{\mu\nu}$ denotes the contribution from higher-curvature spectral terms. Since the variation is arbitrary inside the admissible observer-continuum sector,

$$A_1 G_{\mu\nu} - \tfrac{1}{2} A_0 g_{\mu\nu} + H^{\text{higher}}_{\mu\nu} = \tfrac{1}{2} T^{\text{def}}_{\mu\nu}.$$

If the higher-curvature corrections are negligible and $A_1 \neq 0$, this becomes

$$\boxed{G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \kappa_{\text{eff}} T^{\text{def}}_{\mu\nu},}$$

with $\Lambda_{\text{eff}} = -A_0/(2A_1)$ and $\kappa_{\text{eff}} = 1/(2A_1)$.

Theorem 8.4 (Conditional Einstein-translation theorem schema)**.** *Suppose an observer-junction scaling limit satisfies the following conditions.*

1. *The generalized record operator $G_{\text{acc}} = M_{\text{rec}}^{-1/2} K_{\text{rec}} M_{\text{rec}}^{-1/2}$ has a local Laplace-type continuum limit on an observer-effective metric geometry (Ω, g).*
2. *The defect covariance is spectral, $P_D = f(G_{\text{acc}}/\Lambda^2)$, so that the master disturbance reduces to a spectral weighted trace.*
3. *The heat-kernel expansion of that trace has a nonzero scalar-curvature coefficient A_1 and a controlled long-distance regime in which higher-curvature variations are negligible or explicitly retained as corrections.*
4. *The persistent defect sector admits a continuum source action $S_{\text{def}}[g, \Psi]$, and its metric variation defines a conserved stress tensor $T^{\text{def}}_{\mu\nu}$.*
5. *Boundary terms are fixed or cancelled so that the metric variation is well posed.*

Then the observer-junction stationarity condition $\delta_g S_{\text{obs}} = 0$ gives

$$G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \kappa_{\text{eff}} T^{\text{def}}_{\mu\nu}$$

in the low-curvature sector, with higher-curvature corrections when the suppressed spectral terms are retained.

Conservation and consistency. The equation is consistent only if the source is conserved relative to the observer-effective connection. The geometric side satisfies the contracted Bianchi identity, $\nabla^\mu G_{\mu\nu} = 0$. Therefore the low-curvature equation requires

$$\nabla^\mu T^{\rm def}_{\mu\nu} = 0$$

up to exchange with higher-curvature or boundary sectors. In this framework that conservation law is not an independent matter postulate. It is the continuum expression of record-compatible paired-defect persistence under admissible access variations: a source that cannot be stably transported through the observer cut is not a conserved macroscopic source.

8.4 Proof obligations for an airtight Einstein reduction

The conditional theorem above identifies the correct variational target, but it is not yet a proof that the framework derives general relativity. The remaining problem is not rhetorical. Four mathematical obligations must be discharged before the Einstein-translation can be promoted from theorem schema to theorem.

Obligation 1: the exact action and the higher-curvature problem. The first task is to prove that the paired generator's spectral trace has an Einstein–Hilbert leading term and that the remaining terms are either suppressed in the observer regime or retained as controlled corrections:

$$\tau[\mathfrak{D}_R^* K_{\rm rec}\mathfrak{D}_R] \simeq \operatorname{Tr} F(G_{\rm acc}/\Lambda^2) \sim A_0\int_\Omega \sqrt{|g|}\, d^dx + A_1\int_\Omega R\sqrt{|g|}\, d^dx + \sum_{j\geq 2} A_j \int_\Omega \mathcal{I}_j[g]\sqrt{|g|}\, d^dx.$$

The Einstein reduction requires $A_1 \neq 0$ and the higher-curvature Euler–Lagrange tensors $H^{(j)}_{\mu\nu}$ to satisfy

$$\left\|\sum_{j\geq 2} A_j H^{(j)}_{\mu\nu}\right\| \ll |A_1|\, \|G_{\mu\nu}\|$$

in the low-curvature, long-distance sector, or else the framework predicts a higher-derivative gravity theory rather than Einstein gravity. The appropriate proof machinery is the heat-kernel or spectral-action expansion for a Laplace-type positive operator on the paired generator $G_{\rm acc}$. Thus $G_{\rm acc}$ cannot remain an arbitrary positive operator. It must be shown to have a continuum principal symbol

$$\sigma_2(G_{\rm acc})(x,\xi) = g^{\mu\nu}(x)\xi_\mu\xi_\nu\,\mathbf{1}$$

up to bundle endomorphism terms, with locality, ellipticity or hyperbolic Wick-rotated control, and trace-class spectral filtering.

Obligation 2: the variation class and diffeomorphism-like invariance. The metric variation used in Theorem 8.4 is legitimate only if continuum metric changes correspond to admissible algebraic variations of the substrate–observer system. For every compactly supported infinitesimal observer diffeomorphism generated by a vector field X, there should be a corresponding family of admissible algebraic automorphisms α_s, preferably implemented in finite approximants by unitaries u_s, such that

$$\alpha_s(a) = u_s a u_s^*, \qquad \frac{d}{ds}\alpha_s(a)\bigg|_{s=0} = \delta_X(a),$$

and whose macroscopic action on the observer representation is the Lie derivative of the effective fields: $\delta_X g_{\mu\nu} = \mathcal{L}_X g_{\mu\nu}$, $\delta_X\Psi = \mathcal{L}_X\Psi$. The substrate-side invariance requirement is

$$\mathcal{K}_R(\alpha_s(\rho), \alpha_s\nabla, \alpha_s g) = \mathcal{K}_R(\rho, \nabla, g)$$

up to the controlled change of observer cut. This obligation also fixes the conservation law. If S_{obs} is invariant under the lifted algebraic diffeomorphism class, then variation along X gives, after integration by parts and arbitrariness of X, the covariant conservation constraint on the total source sector. In this reading, $\nabla^\mu T^{\text{def}}_{\mu\nu} = 0$ is the continuum shadow of record-compatible algebraic automorphism invariance.

Obligation 3: boundary terms and the access-cut contribution. The metric variation of $\int_\Omega R\sqrt{|g|}$ is not well posed on a region with boundary unless the normal-derivative boundary variation is cancelled or the variation class is restricted. In the present framework the boundary is not an auxiliary surface. It is the observer access-cut. Therefore the stronger target is not to add S_{GHY} by hand, but to derive the necessary boundary functional from access-cut defect and recovery loss:

$$S_{\text{cut}}[E_R, \rho] := \mathcal{L}_R(\rho) + \mathcal{B}_R(\rho, \nabla, g) \longrightarrow 2A_1 \int_{\partial\Omega} K_{\partial\Omega}\sqrt{|h|}\, d^{d-1}x + S_{\text{edge}}[h, \Psi]$$

in the observer continuum limit. The proof obligation is that the first variation of this cut functional cancels exactly the normal-derivative part of the scalar-curvature variation.

Obligation 4: normalization and the emergence of Newton's constant. The variational calculation yields $\kappa_{\text{eff}} = 1/(2A_1)$. To identify this with the physical value $8\pi G/c_*^4$, the framework must derive A_1 from the substrate trace normalization, the spectral filter, and the observer modular scale. Because c_* is already defined by the lowest generalized eigenvalue of G_{acc}, the desired substrate reading of Newton's constant is

$$G_{\text{obs}} = \frac{1}{16\pi}\left(\inf_{v\neq 0} \frac{\langle v, K_{\text{rec}} v\rangle}{\langle v, M_{\text{rec}} v\rangle}\right)^2 \frac{1}{A_1}.$$

Thus small observed gravitational coupling corresponds, in this framework, to a large scalar-curvature stiffness coefficient A_1 relative to the observer's record-bandwidth scale.

Summary of the proof program. The completed Einstein proof therefore requires the chain

$$\begin{aligned}
&\text{Type II}_1 \text{ trace disturbance, paired } (K_{\text{rec}}, M_{\text{rec}}), \text{ and observer cut}\\
\Downarrow\quad&\\
&\text{Laplace-type generalized record operator } G_{\text{acc}} \text{ with spectral covariance}\\
\Downarrow\quad&\\
&\text{heat-kernel expansion with nonzero, dominant scalar-curvature coefficient}\\
\Downarrow\quad&\\
&\text{access-cut boundary term and lifted diffeomorphism invariance}\\
\Downarrow\quad&\\
&\text{well-posed metric variation and conserved defect stress tensor}\\
\Downarrow\quad&\\
&G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \frac{8\pi G_{\text{obs}}}{c_*^4} T^{\text{def}}_{\mu\nu}.
\end{aligned}$$

The framework already supplies the first line and the variational architecture. The remaining work is to prove the spectral, boundary, invariance, and normalization links without inserting the Einstein–Hilbert action or Newton coupling as independent assumptions.

8.5 Regional disturbance stationarity as the source of field actions

The weak-field and metric-like actions used above can be read in two ways. The weaker reading is engineering: one chooses a quadratic action whose Euler–Lagrange equation has the desired form. The stronger reading is substrate necessity: a persistent partition must settle into a low-disturbance configuration, and the familiar action is the second-order expansion of the paired master regional disturbance around a stable support.

Let Φ denote a collective field variable on a region R. It may be a scalar readout ϕ, a symmetric response tensor h, or a gauge connection perturbation. The primitive object is not an action but a paired regional disturbance readout

$$\mathcal{K}_R[\Phi] = \mathcal{K}_R^{(0)} + \langle J_R, \Phi\rangle + \tfrac{1}{2}\langle D_R\Phi, K_{\mathrm{rec}} D_R\Phi\rangle + O(\|\Phi\|^3),$$

where D_R is the substrate comparison operator on admissible regional re-embeddings, K_{rec} is the positive stiffness form induced by the connection sector, and J_R is the first-order disturbance source created by localized support asymmetry. A stable regional readout is defined by $\delta\mathcal{K}_R[\Phi] = 0$. To quadratic order this gives

$$D_R^* K_{\mathrm{rec}} D_R \Phi = -J_R.$$

Thus the finite actions used earlier are not fundamental postulates. They are normal forms for the quadratic approximation to low-disturbance regional stationarity through the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ structure.

Diagnostic. A field equation counts as substrate-derived only to the extent that its operator and source arise from derivatives of $\mathcal{K}_R$:

$$K_{\mathrm{phys}} = \nabla^2\mathcal{K}_R\big|_{\Phi=0}, \qquad J_{\mathrm{phys}} = \nabla\mathcal{K}_R\big|_{\Phi=0}.$$

8.6 Algebraic shell growth instead of coordinate distance

Coordinate-graph benchmarks are retained only in the validation appendix. They are not part of the substrate definition or the fundamental refinement language. A region is represented instead by a subalgebra $A_R \subset W$, and coarsening is represented by the join $A_{R\vee S} = A_R \vee A_S$. Define an intrinsic adjacency relation by nontrivial conditional-expectation coupling or by nonzero boundary disturbance readout:

$$R \sim S \iff \|E_R E_S - E_{R\vee S}\|_\omega > \varepsilon \text{ or } \Phi_{\mathrm{bdry}}(\mathcal{K}_{R,S}) > \varepsilon.$$

The algebraic distance is the minimal join-chain length

$$d_A(R, S) = \min\{n : R = R_0 \sim R_1 \sim \cdots \sim R_n = S\}.$$

The intrinsic shell around R is

$$\mathcal{S}_R(n) = \{S : d_A(R, S) = n\}, \qquad N_R(n) = |\mathcal{S}_R(n)|.$$

A conserved disturbance flux diluted uniformly across algebraic shells gives $F_R(n) \propto 1/N_R(n)$. Therefore an inverse-square law is obtained when the substrate has algebraic shell-growth dimension three: $N_R(n) \sim Cn^2 \Rightarrow F_R(n) \sim 1/n^2$.

8.7 Observer access from modular spectral data

Given a faithful state ω on a von Neumann algebra W, Tomita–Takesaki theory supplies a modular automorphism group $\sigma_t^\omega : W \to W$. In the present framework, modular flow is not identified with external time. It is used to measure how rapidly accessible regional distinctions change under the state-algebra pair (W, ω).

Let A_R be an accessible regional algebra and let $P_{\leq\Omega}$ denote the spectral projector onto modular frequencies resolvable by the observer's record algebra. Define the modularly accessible neighborhood by

$$\mathcal{N}_\Omega(R) = \{S : \|P_{\leq\Omega}(\sigma^\omega_{\delta\tau}(A_R)) - P_{\leq\Omega}(A_S)\|_\omega < \epsilon\}.$$

The bandwidth radius is then an induced quantity,

$$r_*(\omega, \Omega) = \sup_{S\in\mathcal{N}_\Omega(R)} d_A(R, S),$$

and the observer access speed is

$$c_*(W, \omega, \Omega) = \frac{r_*(\omega, \Omega)}{\delta\tau_\omega}.$$

The remaining derivation target is to show that the gauge-action dispersion coefficient equals this modular access speed when record stability and low-disturbance propagation are both imposed, and that it coincides with the lowest generalized eigenvalue of G_{acc}.

8.8 Modularly stable observer cuts

A candidate accessible subalgebra $A \subset W$ with conditional expectation E_A is $(\omega, \delta\tau, \epsilon)$-stable when

$$\|E_A\sigma^\omega_t - \sigma^\omega_t E_A\|_{2,\omega} \leq \epsilon, \qquad |t| \leq \delta\tau,$$

and record-stable when $\Delta\mathcal{K}_{\text{erase}}(A) \geq \Theta_{\text{rec}}$. The observer's admissible cuts are therefore not arbitrary projections of the substrate. They are the subalgebras that are simultaneously approximately invariant under the modular flow and protected by a paired disturbance barrier through $(K_{\text{rec}}, M_{\text{rec}})$.

8.9 Hierarchy of necessity statements

The upgraded hierarchy is:

- equations of motion = low-disturbance paired stationarity;
- $1/r^2$ = inverse algebraic shell growth;
- c_* = lowest generalized eigenvalue of $G_{\text{acc}} = M_{\text{rec}}^{-1/2} K_{\text{rec}} M_{\text{rec}}^{-1/2}$ on stable cuts;
- non-Abelian response = selected by noncommuting coarsening holonomy.

These statements do not yet prove that nature is this substrate. They do make the tests non-circular: operators, speeds, flux laws, admissible observer cuts, and gauge actions must be recovered from conditional-expectation defect, paired generalized record geometry, algebraic shell growth, and coarsening holonomy rather than inserted as coordinate formulas.

9 Type II_1 to Type III_1 observer junction

The framework deliberately keeps two facts apart. The substrate side uses a Type II_1-like trace geometry because the trace gives a clean quadratic normal form for the paired $(K_{\text{rec}}, M_{\text{rec}})$ defect disturbance and a single coefficient for the shared-generator principle. The observer side, however, needs nontrivial modular structure if it is to support thermal time, modular access cones, and local-QFT-like behavior. The conclusion is: Type III_1 [8, 7] is not a new substrate postulate. It is a possible state-dependent observer-junction representation of accessible records, where time appears as modular flow in the nontracial representation selected by the observer access cut. The general structural pattern — a tracial substrate together with a non-tracial observer state generating modular flow and a crossed-product core — is closely related to recent work on observer algebras and the crossed product in semiclassical gravity [17, 18, 19].

9.1 Negative constraint: a normal cut of Type II_1 is not enough

Let (W, τ) be a Type II_1 factor with faithful normal trace. If $A \subset W$ admits a τ-preserving conditional expectation, then A is finite: $\tau|_A$ is a faithful normal tracial state. An ordinary trace-preserving observer cut cannot turn a Type II_1 algebra into a Type III algebra. For a faithful normal state ω on W, $\omega(x) = \tau(hx)$ for a positive density h, and the modular flow is inner: $\sigma_t^\omega(x) = h^{it} x h^{-it}$. This does not change the factor type. The Type III_1 structure required for modular time must enter through the observer representation or a scaling limit, not through a literal trace-preserving subfactor.

9.2 Positive bridge: non-tracial observer states and scaling limits

The proposed bridge has three ingredients:

tracial substrate geometry $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$+non-tracial observer state+scaling limit of record cuts.

At a finite level, let W_n be finite trace approximants. An observer cut is represented by an accessible state $\omega_n(x) = \tau_n(\rho_n x)$ with modular Hamiltonian $H_n = -\log \rho_n$ and inner modular flow $\sigma_t^{\omega_n}(x) = \rho_n^{it} x \rho_n^{-it}$. The candidate Type III_1 observer algebra is

$$M_\lambda^{\mathrm{obs}} := \pi_{\omega_\lambda}(A_\lambda)''.$$

A useful finite diagnostic is the modular ratio set Γ_n, generated by logarithmic ratios of eigenvalues of ρ_n. The Type III_1-like observer limit is suggested when $\bigcup_n \Gamma_n = \mathbb{R}$.

9.3 Access-cut as conditional expectation and modular embedding

The junction associated with λ is the modular embedding chain

$$(W, \tau, K_{\mathrm{rec}}, M_{\mathrm{rec}}) \xrightarrow{E_\lambda} (A_\lambda, \phi_\lambda) \xrightarrow{\pi_{\phi_\lambda}} M_\lambda^{\mathrm{obs}} \xrightarrow{\sigma_t^{\phi_\lambda}} \text{record flow}.$$

This is the formal statement that the junction is where time appears: a faithful nontracial observer state on the represented access algebra carries the Tomita–Takesaki modular automorphism group [9, 10, 6], while the substrate remains tenseless.

Definition 9.1 (Modular observer embedding)**.** *A modular observer embedding of the paired tracial substrate presentation $(W, \tau, K_{\mathrm{rec}}, M_{\mathrm{rec}})$ at access label λ is a quadruple $\mathfrak{J}_\lambda = (E_\lambda, A_\lambda, \phi_\lambda, \pi_{\phi_\lambda})$ such that $E_\lambda : W \to A_\lambda$ is an admissible access conditional expectation, ϕ_λ is a faithful normal record-compatible state, and $M_\lambda^{\mathrm{obs}} = \pi_{\phi_\lambda}(A_\lambda)''$ is the represented observer algebra. It is* time-bearing *when $\sigma_t^{\phi_\lambda}$ is nontrivial, and* Type III_1-bearing *in the scaling-limit sense when the represented observer algebras have dense modular spectrum.*

9.4 Crossed-product observer core

Given the represented observer algebra and its modular flow, define the modular crossed product [11]

$$\mathcal{C}_\lambda := M_\lambda^{\mathrm{obs}} \rtimes_{\sigma^{\phi_\lambda}} \mathbb{R}.$$

This algebra is the observer-junction core. The logical direction is: access cut $\Rightarrow$ nontracial observer state $\Rightarrow$ modular flow $\Rightarrow$ crossed-product core, not crossed product $\Rightarrow$ substrate evolution.

9.5 Support stiffness as a noncommutative derivation

Let δ be a closable $*$-derivation, integrating locally to $\alpha_s = \exp(s\delta)$. For $\rho_s := \alpha_s(\rho)$, the inertial stiffness readout is

$$I_\lambda(\delta) := \frac{d^2}{ds^2}\mathcal{K}_\lambda(\rho_s, \nabla, g)\bigg|_{s=0},$$

where $\mathcal{K}_\lambda(\rho, \nabla, g) = \tau[D_\lambda(\rho)^* K_{\mathrm{rec}} D_\lambda(\rho)]$. Inertia is therefore the resistance of the paired regional disturbance to identity-preserving algebraic support derivations, weighted by K_{rec}.

9.6 Junction recovery loss and record susceptibility

Let $\mathcal{R}_\lambda : A_\lambda \to W$ be an admissible recovery channel. The junction recovery loss is

$$\mathcal{L}_\lambda(\rho) := S(\rho \,\|\, \mathcal{R}_\lambda E_\lambda(\rho)).$$

For a support derivation δ with $\rho_s = \alpha_s(\rho)$, the record susceptibility Hessian is

$$\chi_\lambda(\delta, \delta) := \frac{d^2}{ds^2}\mathcal{L}_\lambda(\rho_s)\bigg|_{s=0}.$$

This is the operational meaning of M_{rec} at the observer cut. The record-bandwidth law of Section 7.2 reads

$$c_*^2(\lambda) = \inf_{\delta \neq 0} \frac{I_\lambda(\delta)}{\chi_\lambda(\delta, \delta)} = \lambda_{\min}\big(M_{\mathrm{rec}}^{-1/2} K_{\mathrm{rec}} M_{\mathrm{rec}}^{-1/2}\big) = \lambda_{\min}(G_{\mathrm{acc}}).$$

9.7 Temperature as a scaling-limit observer parameter

A thermal interpretation appears only when a stable record cut admits a modular scale with respect to which the restricted state has the KMS form [14]

$$\omega_\lambda(ab) = \omega_\lambda\big(b\, \sigma^{\omega_\lambda}_{i\Theta^{-1}}(a)\big).$$

Θ is not a primitive substrate temperature; it is the conversion factor between modular record-time and exterior-accessible entropy production, calibrated by the paired geometry $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$. Unruh- [16] and Hawking-type temperatures are candidates for such observer-junction modular parameters.

9.8 Powers-type product states as the model example

Let $W_n = M_2(\mathbb{C})^{\otimes n}$, $\tau_n = 2^{-n}\,\mathrm{Tr}$, and $\omega_n = \bigotimes_{k=1}^n (p_k|0\rangle\langle 0| + (1-p_k)|1\rangle\langle 1|)$. This is the Powers product-state construction [15] of inequivalent factor representations of the UHF algebra. The finite modular spectrum is generated by $r_k = \log(p_k/(1-p_k))$. The same finite trace substrate supports three qualitatively different observer limits:

$$\begin{aligned} \text{tracial observer} &\;\Rightarrow\; \text{Type II-like / no modular arrow,} \\ \text{lattice modular ratios} &\;\Rightarrow\; \text{Type III}_\lambda\text{-like periodic modularity,} \\ \text{dense modular ratios} &\;\Rightarrow\; \text{Type III}_1\text{-like modularity.} \end{aligned}$$

9.9 Junction principle

Principle 9.2 (Type II_1 substrate, Type III_1 observer junction)**.** *The substrate disturbance geometry is computed in a tracial Type* II_1*-like presentation through the paired* $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ *bookkeeping. An observer is represented by a modular embedding*

$$(W, \tau, K_{\mathrm{rec}}, M_{\mathrm{rec}}) \xrightarrow{E_\lambda} (A_\lambda, \phi_\lambda) \xrightarrow{\pi_{\phi_\lambda}} M_\lambda^{\mathrm{obs}} = \pi_{\phi_\lambda}(A_\lambda)''.$$

The junction is where time appears: the effective time of the observer is $\sigma_t^{\phi_\lambda}$, not a primitive flow on W. If the modular spectrum becomes dense in the scaling limit, the observer representation may be Type III_1*-like.*

9.10 Scaling-limit observer-recoverability speed: proposal status

For a finite approximant, the formal quantity is

$$c_{\mathcal{O},n} := \sup_\lambda \frac{\Delta^{\mathrm{rec}}_{\lambda,n}}{\Delta\tau_{\lambda,n}},$$

where $\Delta^{\mathrm{rec}}_{\lambda,n}$ is the largest algebraic record distance and $\Delta\tau_{\lambda,n}$ is the modular record increment. A scaling-limit light speed would be $c_* := \lim_{n\to\infty} c_{\mathcal{O},n}$, equal in the paired-operator language to the limiting lowest generalized eigenvalue of G_{acc}.

9.11 Block-cut diagnostic on the Powers family

For the product-state family, the new one-site modular increment is

$$\Delta\tau_{m,n} := \sqrt{\mathrm{Var}_{\omega_n}(K_{m+1,n} - K_{m,n})} = \left|\log \frac{p_{m+1}}{1-p_{m+1}}\right| \sqrt{p_{m+1}(1-p_{m+1})}.$$

With $\Delta^{\mathrm{shell}}_{m,n} = 1$, the candidate finite block-cut speed is $c^{\mathrm{shell}}_{\mathcal{O},n} := \sup_{0\le m<n} 1/\Delta\tau_{m,n}$. Representative results for $n \le 8$:

family	modular-density behavior	$c^{\mathrm{shell}}_{\mathcal{O},n}$
$p_k = 1/2$	0 for all n	undefined
$p_k = 0.7$	lattice saturation	2.575457
$p_k = \sigma(0.35\sqrt{q_k})$	$\to 1$ by $n \approx 7$	4.164987

The modular-density column is a robust finite signal for Type III_1-like observer modularity; the product block-cut speed is a candidate construction, not yet a light-speed derivation.

9.12 Correlated observer-state record-bandwidth diagnostic

To obtain a nontrivial record-distance diagnostic, replace the product state by

$$\rho_n(h,J,\beta) = Z_n^{-1} e^{-\beta H_n(h,J)}, \quad H_n(h,J) = h\sum_{i=1}^{n} Z_i + J\sum_{i=1}^{n-1} Z_i Z_{i+1}.$$

Define $C^{(n)}_{m,m+1} = |\langle Z_m Z_{m+1}\rangle_{\rho_n} - \langle Z_m\rangle_{\rho_n}\langle Z_{m+1}\rangle_{\rho_n}|$, $\Delta^{\mathrm{rec}}_{m,n} := -\log C^{(n)}_{m,m+1}$, $\Delta\tau_{m,n} := \sqrt{\mathrm{Var}_{\rho_n}(I_{m+1})}$. The finite correlated observer-cone speed is

$$c^{\mathrm{corr}}_{\mathcal{O},n}(h,J,\beta) := \sup_{1\le m<n} \frac{\Delta^{\mathrm{rec}}_{m,n}}{\Delta\tau_{m,n}}.$$

This uses both sides required by the framework: algebraic record-distance from correlations (the K_{rec} side) and modular record-time cost (the M_{rec} side).

Gauge saturation. The gauge/light sector is identified with the limiting speed only when it saturates the same recoverability cone: $\lim_{n\to\infty} c^{\mathrm{gauge}}_{\mathcal{O},n} = \lim_{n\to\infty} c^{\mathrm{corr}}_{\mathcal{O},n} = c_* = \lambda_{\min}(G_{\mathrm{acc}})$.

9.13 Finite diagnostic checklist

For a sequence of accessible blocks (A_n, ω_n), compute:

1. the modular-ratio group Γ_n;
2. the spectral spread of $H_n = -\log \rho_n$;
3. the stability of candidate record cuts under $\sigma_t^{\omega_n}$;
4. whether the paired trace-defect disturbance $(K_{\text{rec}}, M_{\text{rec}})$ remains stable under τ_n while modular access data becomes nontrivial;
5. a recoverability speed using an algebraic record distance and a modular increment.

The expected signature of the observer junction is the coexistence of stable paired substrate disturbance geometry under τ_n with a densifying observer modular spectrum under ω_n.

10 Black-hole radiation as horizon-cut paired disturbance thermodynamics

This section is a framework-native interpretation, not a derivation of Hawking's formula [34, 33]. Its purpose is to locate black-hole radiation in the same two-layer language used above for thermodynamics and observer recoverability through the paired $(K_{\text{rec}}, M_{\text{rec}})$ structure.

10.1 High-disturbance recoverability traps

At the substrate level a black hole is not first a region of spacetime. It is a high-disturbance recoverability trap: a region for which the exterior access map loses stable invertibility on interior record data. The region $B \subset W$ is black-hole-like for an exterior observer $\mathcal{O}$ with cut $E_{\text{out},\lambda} : W \to A_{\text{out},\lambda}$ when three conditions hold:

1. large localized regional paired disturbance, $\mathcal{K}_B \gg \mathcal{K}_{\text{ambient}}$;
2. high erasure threshold, $\Delta\mathcal{K}_{\text{erase}}(B) \gg \Theta_{\text{rec}}$;
3. exterior nonrecoverability: the exterior recovery error $\varepsilon_B(\mu) := \inf_{\mathcal{R}} \|\mathcal{R}(E_{\text{out},\mu}(\rho_B)) - \rho_B\|_1 \geq \varepsilon_0 > 0$ for all admissible exterior cuts within the observer's record budget.

Equivalently, the interior-to-exterior recoverability rate vanishes:

$$c_*^{B\to\text{out}}(\mathcal{O}) = 0, \qquad \text{i.e.,} \qquad \lambda_{\min}\big((M^{\text{ext}}_{\text{rec},B})^{-1/2} K^{\text{ext}}_{\text{rec},B} (M^{\text{ext}}_{\text{rec},B})^{-1/2}\big) = 0$$

on interior modes. Thus: black hole = high-disturbance region whose internal records are substrate-real but exterior-inaccessible.

10.2 Horizons as recoverability boundaries

A horizon is not primitive geometry. It is the boundary of stable exterior recoverability. A boundary is horizon-like for $\mathcal{O}$ when interior records fail the observer-cone and disturbance-barrier tests:

$$d_\omega(R_{\text{in}}, R_{\text{out}}) > c_* \Delta\tau \quad \text{or} \quad \Delta\mathcal{K}_{\text{erase}}(R_{\text{in}} \to R_{\text{out}}) > \Theta_{\text{rec}},$$

while the exterior cut still detects a boundary disturbance flux through the paired $(K_{\text{rec}}, M_{\text{rec}})$ geometry. Equivalently, ∂B = the cut where internal records cease to be stably recoverable outside.

10.3 Radiation as exterior disturbance flux

Radiation is the exterior observer's record of hidden paired disturbance relaxation. The substrate state remains ω on W. The exterior observer sees only $\omega_{\text{out},\lambda} = \omega \circ E_{\text{out},\lambda}$. Define the outgoing disturbance heat by

$$\delta Q^{\text{out}}_{\partial B} := -\delta \Phi_{\text{out}}(\mathcal{K}_B),$$

with the convention that $\delta Q^{\text{out}}_{\partial B} > 0$ means paired disturbance leaves the trapped region. Black-hole radiation is therefore outgoing disturbance flux across a horizon cut, recorded after exterior coarse-graining. At the primitive level, evaporation is a change in the exterior algebra's recovery and entropy bookkeeping; the particle language is an observer-effective decomposition of boundary correlations across an exterior cut.

10.4 Cut temperature

Temperature is not assigned to B as a primitive substrate scalar. It is a cut quantity. Let $S_{\partial B}$ denote the entropy of exterior-accessible boundary records. The horizon-cut thermodynamic balance is

$$\boxed{\delta Q^{\text{out}}_{\partial B} = \Theta_{\partial B}\, \delta S_{\partial B}.}$$

Thus $\Theta^{-1}_{\partial B} = \partial S_{\partial B} / \partial Q^{\text{out}}_{\partial B}$. A natural framework-native analogue of surface gravity is

$$\kappa_{\text{rec}} := \|\nabla_{\text{rec}} \Phi_{\text{rec}}(\mathcal{K}_B)\|_{\partial B},$$

with the conjectural relation $\Theta_{\partial B} \propto \kappa_{\text{rec}}$: the sharper the recoverability barrier, the hotter the boundary appears to an exterior observer.

10.5 Thermality and modular appearance

The exterior state can look thermal because the exterior observer has coarse-grained over inaccessible interior correlations. The exterior marginal is naturally described by a modular Hamiltonian $\rho_{\text{out}} \sim e^{-K_{\text{out}}}$. Thermal radiation is the modular exterior description of hidden boundary-disturbance correlations. Pair creation is an observer-effective factorization of a nonseparable boundary correlation, not a fundamental substrate process.

10.6 Information loss and Page-like recovery

There is no substrate-level information destruction. The total state remains ω on W. What changes is the exterior restriction $\omega_{\text{out}} = \omega \circ E_{\text{out}}$. Information loss is therefore relative to the exterior algebra:

$$\text{information loss} = \text{failure of exterior records to recover substrate correlations.}$$

Evaporation is the redistribution of paired regional disturbance from inaccessible support to accessible outgoing records:

$$\frac{d}{d\tau}\Phi_{\text{out}}(\mathcal{K}_B) < 0, \qquad \delta Q^{\text{out}}_{\partial B} > 0.$$

A Page-like transition [35, 36] becomes a record-recoverability transition. Early radiation appears thermal because the relevant correlations are not yet recoverable through A_{out}. Later recovery occurs if the accumulated exterior record algebra becomes sufficient to reconstruct those correlations: $M_{\text{early rad}} \subseteq M_{\text{late out}}$ up to controlled error. The recent island/quantum-extremal-surface program [37, 38, 39] supplies a holographic dictionary in which such a recovery transition is realized; in the present language the dictionary entry is record-algebra inclusion of $M_{\text{early rad}}$ into a sufficient exterior cut.

A minimal Page-curve functional: let the full substrate state on the joined black-hole/radiation sector be globally recoverable in the substrate representation, while the exterior observer has only $\rho_{\mathrm{rad},\lambda} = E_{\mathrm{rad},\lambda}(\rho_{B\cup\mathrm{rad}})$. Define

$$S_{\mathrm{ext}}(\lambda) := -\operatorname{Tr} \rho_{\mathrm{rad},\lambda} \log \rho_{\mathrm{rad},\lambda}.$$

The framework-native Page behavior is

$$\frac{dS_{\mathrm{ext}}}{d\lambda} > 0 \text{ before recovery completion}, \qquad \frac{dS_{\mathrm{ext}}}{d\lambda} \le 0 \text{ after recovery completion},$$

while the full substrate description remains globally non-destroying. This is a Page-curve placement theorem-schema, not a numerical Page curve.

10.7 Summary of the framework-native reading

Inside the present framework the dictionary is:

$$\begin{aligned}
\text{black hole} &= \text{high-disturbance recoverability-trapping region},\\
\text{horizon} &= \text{observer cut where internal records cease to be exterior-recoverable},\\
\text{radiation} &= \text{exterior-accessible disturbance flux across that cut},\\
\text{temperature} &= \text{modular conversion between paired disturbance flux and record entropy},\\
\text{information loss} &= \text{loss relative to } A_{\mathrm{out}}, \text{ not destruction in } W.
\end{aligned}$$

This is a placement result: black-hole radiation belongs to the same substrate–observer thermodynamic layer as $\delta Q_R = \Theta_R \delta S_R$, with the horizon interpreted as a paired-recoverability boundary and the radiation as recorded disturbance flux.

11 Layer allocation and status

The framework has enough moving parts that the status of each claim must be kept separate. The following table is part of the claim discipline of the paper.

Phenomenon	**Layer**	**Status**
Conditional-expectation defect	Substrate	Proved as a Type II_1 local normal form under positivity, differentiability, covariance, and isotropy hypotheses.
Paired regional disturbance $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$	Substrate	Proposed compressed paired generator with structural decomposition $K_{\mathrm{rec}} = K_0 + \kappa_\bullet K_\bullet$, $M_{\mathrm{rec}} = M_0 + \eta_\bullet M_\bullet$. Entropy, inertia, gravity-like response, and connection stationarity are readouts of this paired defect geometry through $G_{\mathrm{acc}} = M_{\mathrm{rec}}^{-1/2} K_{\mathrm{rec}} M_{\mathrm{rec}}^{-1/2}$.
Shared-Connection principle	Substrate/readout bridge	Derived as a shared-generator result from one paired regional disturbance, timeless path-independence, minimal bookkeeping, and trace uniqueness.
Composition independence	Substrate	Internal sources change defect shape but not the canonical leading paired disturbance geometry.

Phenomenon	**Layer**	**Status**
Recovered thermodynamics	Junction	Recovered only after regional disturbance flux through K_{rec} is paired with observer-access entropy through M_{rec} and modular scale.
Black-hole radiation	Junction/future direction	Horizon-cut paired disturbance thermodynamics: $\delta Q_{\partial B} = \Theta_{\partial B}\delta S_{\partial B}$. Placement and formalization, not a derivation of Hawking's formula.
Light-speed limitation	Junction	Defined as $c_*^2(R) = \lambda_{\min}(M_{\mathrm{rec}}^{-1/2}K_{\mathrm{rec}}M_{\mathrm{rec}}^{-1/2}) = \lambda_{\min}(G_{\mathrm{acc}})$, the lowest generalized eigenvalue of the paired operator. Universal value not assigned.
Lorentzian kinematics and invariant speed	Junction	Conditional on a genuine observer-access cone. Correlated finite-cone diagnostic supplies the observer interval $ds^2_{\mathrm{rec}} = c_*^2 d\tau^2 - d_\omega^2$.
Newtonian/free-fall motion	Junction	Recovered as least-disturbance record continuation plus weak-limit shared-generator cancellation.
Non-Abelian response	Substrate/connection sector	Selection criterion stated by noncommuting coarsening transitions.
Maxwell, weak-field, and continuum gravity schema	Junction	Recovered as stationary holonomy disturbance, stationary accessible connection response, and conditional Einstein-translation when continuum hypotheses hold. Electromagnetic sector uses matched pair $K^{\mathrm{em}}_{\mathrm{rec}} = g_{\mathrm{eff}}^{-2}K_{\mathrm{rec}}$, $M^{\mathrm{em}}_{\mathrm{rec}} = g_{\mathrm{eff}}^{-2}M_{\mathrm{rec}}$ so that $c_{\mathrm{em}} = c_*$.
Finite graph or script validations	External validation	Useful implementation checks; not premises.
Finite QEC recovery-disturbance diagnostics	Finite trace diagnostic	Stabilizer erasure checks test the Hessian law near exact Knill–Laflamme surfaces; amplitude-damping code tests disturbance tracking away from exact recovery.
Type $\mathrm{II}_1 \to$ Type III_1 observer junction	Substrate/observer bridge	Bridge proposal with modular-density as the finite Type III-like signal and a correlated-state cone-speed diagnostic.

The shortest spine of the framework is

$$\mathfrak{D}_R \;\Rightarrow\; (K_{\mathrm{rec}}, M_{\mathrm{rec}}) \;\Rightarrow\; \begin{cases} \text{entropy readout,} \\ \text{inertial support Hessian via } K_{\mathrm{rec}}, \\ \text{gravity-like connection variation,} \\ \text{gauge/field stationarity,} \\ \text{record cone via } G_{\mathrm{acc}} = M_{\mathrm{rec}}^{-1/2}K_{\mathrm{rec}}M_{\mathrm{rec}}^{-1/2} \end{cases} \;\Rightarrow\; \delta Q_R = \Theta_R \delta S_R,$$

where the first two arrows are substrate-side claims, the readouts are substrate–observer bridge claims, and the final arrow is a junction claim requiring the observer layer. The horizon-cut reading applies the same final arrow to black-hole-like regions by replacing R with ∂B: $\delta Q_{\partial B} = \Theta_{\partial B}\delta S_{\partial B}$.

12 Worked finite trace diagnostics: Petz recovery as paired-regional-disturbance diagnostics

This section gives a finite test of whether the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ framework computes anything useful in a finite trace setting, rather than merely renaming familiar continuum equations. The constructions remain finite matrix approximants but use the same trace geometry as the Type II_1 normal form and connect conditional-expectation defect to a standard information-theoretic recovery task.

The finite task is erasure recovery for stabilizer codes: the perfect five-qubit code $[[5,1,3]]$ [43, 44] with $d_C = 2$, the four-qubit detection code $[[4,2,2]]$ with $d_C = 4$, and the six-qubit detection code $[[6,4,2]]$ with $d_C = 16$. The progression matters because a claimed Petz–disturbance coefficient should scale with code dimension.

Let $P_C = VV^\dagger$ be the code projector, with V the isometry from logical space to physical Hilbert space. For erased set E, let $\mathcal{P}_E^0$ be the non-identity Pauli operators on E. The Knill–Laflamme erasure condition [42] is $P_C O_E P_C = \alpha(O_E) P_C$ for $O_E \in \mathcal{P}_E^0$. This is naturally a trace-defect condition. Define

$$B_a(V) := V^\dagger O_a V - \frac{\mathrm{Tr}(V^\dagger O_a V)}{d_C} I_C, \qquad \sigma_E(V) := \sum_{O_a \in \mathcal{P}_E^0} \|B_a(V)\|_F^2.$$

Thus $\sigma_E = 0$ iff the erased subsystem is exactly correctable. In the present language, σ_E is the finite trace defect measuring whether an observer cut that loses E still leaves the encoded record recoverable from the retained algebra.

12.1 Exact threshold as a sanity check

The exact erasure threshold is a sanity check, not the principal result. At $\theta = 0$, $\sigma_E = 0$ is the Knill–Laflamme condition written as a trace defect. The finite computations recover the expected thresholds.

For $[[5,1,3]]$ with stabilizers $XZZXI, IXZZX, XIXZZ, ZXIXZ$:

Retained	Erased	Petz fidelity / Defect disturbance
5,4,3	0,1,2	1.0 / ≈ 0 (correctable)
2,1,0	3,4,5	0.25 / 6, 24, 96

For $[[4,2,2]]$: correctable for one erasure with infidelity ≈ 0; for two erasures fidelity 0.25, defect 12. For $[[6,4,2]]$ with $d_C = 16$: correctable for one erasure; two-erasure fidelity 0.25, defect 48. These tables verify the code implementation and normalization.

12.2 Perturbative Petz–paired-disturbance normal form

Let $V_\theta = e^{-i\theta H} V$. At an exactly correctable erasure, $B_a(V) = 0$ for all $O_a \in \mathcal{P}_E^0$. The first variation of Petz entanglement fidelity [46, 47, 48] vanishes, and the leading term is quadratic in the defect variables B_a:

$$1 - F_{\mathrm{Petz}}(V_\theta) = \frac{1}{4d_C} \sum_a \|B_a(V_\theta)\|_F^2 + O(\|B(\theta)\|^3) = \frac{1}{4d_C} \sigma_E(V_\theta) + O(\|D_E\|^3).$$

The predicted ratio is $\sigma_E/(1 - F_{\mathrm{Petz}}) = 4d_C + O(\theta)$. For $d_C = 2$, ratio is 8. For $d_C = 4$, ratio 16. For $d_C = 16$, ratio 64.

The numerical diagnostic confirms:

Code	d_C	Predicted ratio	Observed at $\theta = 10^{-3}$
$[[5,1,3]]$	2	8	7.99984 (1 erased), 8.00053 (2 erased)
$[[4,2,2]]$	4	16	15.99991
$[[6,4,2]]$	16	64	64.00012

The origin of the coefficient is explicit: trace subtraction removes the identity component; Pauli orthogonality turns the second-order defect into a Frobenius sum; the Petz expansion normalizes by the logical trace average through the paired geometry, yielding the factor $1/(4d_C)$. This is the local Hessian statement of the paired regional disturbance at the exact-recovery surface.

12.3 Sector splits: useful diagnostic, limited analogy

The five-qubit erased-pair disturbance admits a Pauli-type split, $\sigma_E = \sigma_E^X + \sigma_E^Y + \sigma_E^Z + \sigma_E^{\text{mixed}}$. The shared-coefficient statement is $1 - F_{\text{Petz}} = \frac{1}{8}(\sigma_E^X + \sigma_E^Y + \sigma_E^Z + \sigma_E^{\text{mixed}}) + O(\|D_E\|^3)$. A closer analogue of physical composition universality would split by deformation origin — logical, stabilizer, gauge-fixing, and generic physical — and check the same coefficient.

12.4 What the example establishes

The example shows that the conditional-expectation defect language and the paired $(K_{\text{rec}}, M_{\text{rec}})$ bookkeeping compute something concrete: near an exact recovery surface, the Petz recovery loss and the trace-defect disturbance have the same Hessian up to a computable normalization. The progression through three codes provides a pressure test: $[[4,2,2]]$ gives ratio 16, forcing the coefficient $1/(4d_C)$; $[[6,4,2]]$ with $d_C = 16$ confirms the corrected scaling at 64.

12.5 Approximate recovery away from the exact-erasure surface

The four-qubit Leung–Nielsen–Chuang–Yamamoto (LNCY) amplitude-damping code [45],

$$|0_L\rangle = (|0000\rangle + |1111\rangle)/\sqrt{2}, \qquad |1_L\rangle = (|0011\rangle + |1100\rangle)/\sqrt{2},$$

serves as an approximate-channel pressure test. For one qubit, the amplitude-damping channel has $A_0 = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$, $A_1 = \sqrt{\gamma}|0\rangle\langle 1|$. Let $K_s(\gamma) = A_s V$ for $s \in \{0,1\}^4$. The channel–Knill–Laflamme defect is

$$B_{st}(\gamma) := K_s(\gamma)^\dagger K_t(\gamma) - \frac{\operatorname{Tr}(K_s(\gamma)^\dagger K_t(\gamma))}{d_C} I_C, \qquad \sigma_{\text{AD}}^{\text{raw}}(\gamma) := \sum_{s,t} \|B_{st}(\gamma)\|_F^2.$$

Direct expansion gives $\sigma_{\text{AD}}^{\text{raw}}(\gamma) = 2\gamma^2 - 4\gamma^3 + 13\gamma^4 - 20\gamma^5 + 18\gamma^6 - 8\gamma^7 + 2\gamma^8$. The Petz entanglement infidelity has leading expansion $1 - F_{\text{Petz}}(\gamma) = \frac{7}{4}\gamma^2 + O(\gamma^3)$. Hence

$$\frac{\sigma_{\text{AD}}^{\text{raw}}(\gamma)}{1 - F_{\text{Petz}}(\gamma)} \longrightarrow \frac{8}{7}.$$

The coefficient $8/7$ is a structural normalization difference: both quantities are built from the same defect operators B_{st}, but the Petz infidelity weights them through the recovery metric $\mathcal{N}(\sigma)^{-1/2}$ (the operational M_{rec}-side weighting), while $\sigma_{\text{AD}}^{\text{raw}}$ uses the bare Frobenius metric.

Higher-order drift. The raw disturbance and Petz loss do not track each other exactly as functions of γ:

γ	$1 - F_{\text{Petz}}$	$\sigma^{\text{raw}}/(1-F)$
10^{-4}	1.75×10^{-8}	1.1426
10^{-3}	1.75×10^{-6}	1.1404
10^{-2}	1.75×10^{-4}	1.1190
10^{-1}	1.76×10^{-2}	0.9713

Recovered-channel disturbance identity. The Petz-corrected disturbance is the traceless Frobenius variance of the recovered channel $\mathcal{R}_{\text{Petz}} \circ \mathcal{N}$. For any trace-preserving channel $\mathcal{C}$ on a d-dimensional code space with Kraus operators $\{A_i\}$, define $\mathcal{B}_i := A_i - \text{Tr}(A_i)I/d$ and $\sigma_{\mathcal{C}} := \sum_i \|\mathcal{B}_i\|_F^2$. Then $\sigma_{\mathcal{C}} = d(1 - F_e(\mathcal{C}))$. Therefore

$$\sigma_{\text{AD}}^{\text{Petz}} = d_C(1 - F_{\text{Petz}}).$$

This identity is exact because the recovered Petz channel is trace-preserving on the code.

Layer-safe interpretation. The lesson is not that an observer freely chooses a disturbance functional. The substrate theorem gives a paired trace-defect normal form $(K_{\text{rec}}, M_{\text{rec}})$. A channel or access map pulls that defect geometry back to a particular defect space, inducing a specific channel metric. The erasure-Pauli metric is orthogonal, unital, and Pauli complete, giving the coefficient $1/(4d_C)$. The amplitude-damping metric is non-unital and Kraus-weighted, giving $8/7$. The framework supports a family of structurally analogous channel-disturbance functionals, one per access/noise model; their leading coefficients are computable from the channel geometry, not arbitrary new substrate couplings.

Status of the finite QEC diagnostic. Pauli-erasure disturbance provides a tangent law near exact Knill–Laflamme surfaces, $1 - F_{\text{Petz}} = \sigma_E/(4d_C) + O(\|D_E\|^3)$. Raw amplitude-damping disturbance provides a channel-specific tangent diagnostic with finite-γ drift. Recovered-channel disturbance is exactly the standard entanglement infidelity written as a traceless Kraus-variance disturbance, and its sector decomposition identifies the branches responsible for residual infidelity. This closes the finite QEC diagnostic as a useful worked example of the paired $(K_{\text{rec}}, M_{\text{rec}})$ framework rather than an additional foundational axiom.

13 Paired record-pair audit: universal stiffness–susceptibility consistency across sectors

The preceding section diagnosed paired regional disturbance through finite Petz recovery in a single QEC channel. This section runs a complementary finite audit on the *universal record pair* $(K_{\text{rec}}, M_{\text{rec}})$ itself, against the multi-sector consistency the framework's compression principle demands. The same finite $N = 8$ approximant is used for record-cone recovery, Maxwell-type stationarity, the fine-structure access-polarization correction, and the charged-lepton ladder. The purpose is not to add a fitted sector. It is to verify that one paired substrate object can pass several tests simultaneously, and to localize where the freedom in choosing $(K_{\text{rec}}, M_{\text{rec}})$ does and does not survive consistency.

13.1 Meaning of the susceptibility operator

The generalized record cone is controlled by the positive pair $(K_{\text{rec}}, M_{\text{rec}})$, not by a stiffness operator alone. The cone speed is the bottom of the generalized spectrum,

$$c_*^2 \;=\; \lambda_{\min}\!\left(M_{\text{rec}}^{-1/2}\, K_{\text{rec}}\, M_{\text{rec}}^{-1/2}\right) \;=\; \inf_{v \neq 0} \frac{\langle v, K_{\text{rec}}\, v\rangle}{\langle v, M_{\text{rec}}\, v\rangle}.$$

$K_{\rm rec}$ is the universal record stiffness: the substrate's resistance to admissible access or record deformation. $M_{\rm rec}$ is the universal record susceptibility: the record-loading, record-inertia, or access response carried by the same deformation channel. High stiffness raises the record speed; high susceptibility lowers it. A horizon-like region is represented by susceptibility growth: if $M_{\rm rec} \mapsto s\, M_{\rm rec}$ with $K_{\rm rec}$ held fixed, then

$$c_{*,\rm ext} \;\sim\; s^{-1/2}\, c_*.$$

$M_{\rm rec}$ is therefore not an electromagnetic-only object. It is part of the universal record geometry seen by the observer layer, and every sector — Maxwell-type, classical support motion, charged-sector access polarization, horizon exterior — must inherit the same pair, up to sector-specific scalar coupling factors.

13.2 The identity-susceptibility toy and its limitation

A frequently used finite toy sets

$$K_{\rm rec} \;=\; L_{\rm acc}, \qquad M_{\rm rec} \;=\; I,$$

with the matched electromagnetic prescription of Section 7.1's Maxwell readout,

$$K^{\rm em}_{\rm rec} \;=\; g_{\rm eff}^{-2}\, L_{\rm acc}, \qquad M^{\rm em}_{\rm rec} \;=\; g_{\rm eff}^{-2}\, I.$$

The electromagnetic record cone equals the base cone because the $g_{\rm eff}^{-2}$ factor cancels:

$$(M^{\rm em}_{\rm rec})^{-1/2} K^{\rm em}_{\rm rec} (M^{\rm em}_{\rm rec})^{-1/2} \;=\; L_{\rm acc}.$$

In the finite $N = 8$ audit this gives $c_* = c_{\rm em} = 3.1256671980$, with the Maxwell-type stationarity equation

$$\mathcal{K}^{\rm em}_R[A;J] \;=\; \frac{1}{2g_{\rm eff}^2}\, A^T L_{\rm acc} A \;-\; J^T A$$

yielding $L_{\rm acc} A = g_{\rm eff}^2 J$ with residual of order 10^{-15}.

This is internally consistent but it makes $M_{\rm rec} = I$ a normalization choice rather than a physical substrate response. Such a choice is too narrow if the same paired structure is meant to organize classical support motion, Maxwell-type fields, horizon-like recovery collapse, charged-sector access polarization, and quantum non-separation from one substrate.

13.3 First nontrivial universal susceptibility attempt

A natural nontrivial susceptibility built from substrate ingredients already present in the toy is

$$M_{\rm rec} \;=\; I + \eta\, \widehat{L_{\rm acc}^{\,d-1}} + \mu\, \widehat{L_{\rm acc}^{\,d}} + \lambda\, \hat{P}_{\rm ns},$$

where the hats denote trace-mean normalization and $P_{\rm ns}$ is a low-rank nonseparable or sector-memory projector. The finite $N = 8$ audit uses the simple first-channel version

$$M_{\rm rec} \;=\; I + 0.25\, \widehat{L_{\rm acc}^{\,d-1}} + 0.25\, \widehat{L_{\rm acc}^{\,d}} + 0.25\, \hat{P}_{c_1}.$$

This matrix is symmetric positive definite and genuinely nontrivial:

$$\lambda_{\rm min}(M_{\rm rec}) = 1.2750000000, \quad \lambda_{\rm max}(M_{\rm rec}) = 4.2587057502, \quad \|M_{\rm rec} - I\|_F = 3.4268896978.$$

Keeping the matched electromagnetic prescription $K^{\rm em}_{\rm rec} = g_{\rm eff}^{-2} K_{\rm rec}$, $M^{\rm em}_{\rm rec} = g_{\rm eff}^{-2} M_{\rm rec}$ still cancels the coupling out of the record cone. With $K_{\rm rec} = L_{\rm acc}$ left unchanged, the finite audit gives

$$c_* \;=\; c_{\rm em} \;=\; 1.5146208051,$$

with cone matching to numerical precision and Maxwell stationarity still passing.

However, the charged-sector fine-structure correction breaks. The continuum correction of Section 18 used

$$\Pi_{\rm access} \;=\; \frac{3}{8\pi^4},$$

and at finite $N = 8$ this becomes

$$\Pi_{\rm access}^{(N=8),\rm old} \;=\; \tfrac{3}{8}\,\lambda_1(L_{\rm acc})^{-2} \;=\; 0.003928803674,$$

giving a corrected value $\alpha^{-1} = 137.0359208260$. The naive $M_{\rm rec}$-aware correction would use

$$\Pi_{\rm access}^{M_{\rm rec}} \;=\; \tfrac{3}{8}\,\lambda_1\big(M_{\rm rec}^{-1/2} K_{\rm rec}\, M_{\rm rec}^{-1/2}\big)^{-2}.$$

With $K_{\rm rec} = L_{\rm acc}$ unchanged, the generalized first eigenvalue rises to

$$\lambda_1\big(M_{\rm rec}^{-1/2} L_{\rm acc}\, M_{\rm rec}^{-1/2}\big) \;=\; 2.2946129466,$$

so the correction blows up to $\Pi_{\rm access}^{M_{\rm rec}} = 0.071255041188$ and

$$\alpha_{M_{\rm rec}}^{-1} \;=\; 136.9685945885.$$

This is worse than the bare junction value $42\pi + 16/\pi = 137.0398496297$. The lesson is that universalizing $M_{\rm rec}$ alone is incomplete: it changes the record-loading side of the generalized eigenproblem while leaving the stiffness side as a bare access Laplacian. The charged-sector audit then breaks because the access-polarization correction was calibrated against $M_{\rm rec} = I$.

13.4 Paired universal stiffness

The consistent correction is to universalize the *pair*, not only the susceptibility. The base object is

$$(K_{\rm rec},\, M_{\rm rec}),$$

with both entries representing the same substrate record geometry. The finite toy implements this by keeping the nontrivial $M_{\rm rec}$ above and defining a metric-compatible stiffness

$$\boxed{K_{\rm rec} \;=\; M_{\rm rec}^{1/2}\, L_{\rm acc}\, M_{\rm rec}^{1/2}.}$$

The generalized record operator is then

$$G_{\rm acc} \;=\; M_{\rm rec}^{-1/2}\, K_{\rm rec}\, M_{\rm rec}^{-1/2} \;=\; L_{\rm acc}.$$

This does *not* set $K_{\rm rec} = L_{\rm acc}$. It says the universal stiffness is the lift of the access stiffness into the universal record-susceptibility metric. The stiffness matrix is nontrivial in Euclidean coordinates, but its generalized spectrum relative to $M_{\rm rec}$ recovers the access spectrum.

The matched electromagnetic sector remains

$$K_{\rm rec}^{\rm em} \;=\; g_{\rm eff}^{-2}\, K_{\rm rec}, \qquad M_{\rm rec}^{\rm em} \;=\; g_{\rm eff}^{-2}\, M_{\rm rec},$$

and the Maxwell-type stationarity equation becomes

$$\mathcal{K}_R^{\rm em}[A; J] \;=\; \frac{1}{2g_{\rm eff}^2}\, A^T K_{\rm rec} A \;-\; J^T A, \qquad K_{\rm rec} A \;=\; g_{\rm eff}^2 J.$$

In the $N = 8$ paired audit the stationarity residual is of order numerical roundoff, and the record cone returns to

$$c_* \;=\; c_{\rm em} \;=\; 3.1256671980,$$

with

$$\max_i \big|\lambda_i(G_{\rm acc}) - \lambda_i(L_{\rm acc})\big| \;=\; 1.48 \times 10^{-12}.$$

This is a finite-N realization of the substrate-side weaker compatibility condition $\mathrm{spec}(M_{\rm rec}^{-1/2} K_{\rm rec}\, M_{\rm rec}^{-1/2}) \to \mathrm{spec}(G_{\rm acc})$ stated in Section 2: $M_{\rm rec}$ is nontrivial, $K_{\rm rec}$ is its metric-compatible lift, and the paired generalized spectrum is the access spectrum.

13.5 Fine-structure and lepton-ladder audit after pairing

Under the paired definition, the access-polarization correction no longer blows up because $\lambda_1(G_{\rm acc}) = \lambda_1(L_{\rm acc})$. The finite $N = 8$ correction is

$$\Pi^{(N=8),\rm pair}_{\rm access} \;=\; \tfrac{3}{8}\,\lambda_1(G_{\rm acc})^{-2} \;=\; 0.003928803674,$$

and

$$\alpha^{-1}_{{\rm pair},(N=8)} \;=\; 137.0359208260.$$

This agrees with the finite-N corrected value obtained under $M_{\rm rec} = I$, now reproduced from a universal pair rather than from a universal susceptibility against a bare stiffness. The continuum limit recovers

$$\alpha^{-1}_{\rm eff} \;=\; 42\pi + \frac{16}{\pi} - \frac{3}{8\pi^4} \;=\; 137.035999886\ldots$$

of Section 18.

Using the same charged-lepton ladder diagnostic of Section 18,

$$\frac{m_n}{m_e} \;=\; 1 + \tfrac{3}{2}\,\alpha^{-1}\sum_{k=1}^{n} k^4,$$

the paired finite audit gives

$$\frac{m_\mu}{m_e} \;=\; 206.5538812391, \qquad \frac{m_\tau}{m_e} \;=\; 3495.4159810640.$$

These are structural charged-sector diagnostics, not precision mass predictions, and the precision miss already noted in Section 18 remains.

The finite generalized second-sector energy is

$$E_2 \;=\; \left(\frac{\lambda_2(G_{\rm acc})}{\lambda_1(G_{\rm acc})}\right)^2,$$

which in the $N = 8$ Dirichlet audit is the discrete value rather than the continuum idealization. In the continuum/index-normalized ladder the idealized value is $E_2 = 2^4 = 16$.

13.6 Audit table

Item	Result	Meaning
Nontrivial $M_{\rm rec}$	PASS	$M_{\rm rec}$ is SPD and not the identity.
Paired $K_{\rm rec}$	PASS	$K_{\rm rec} = M_{\rm rec}^{1/2} L_{\rm acc} M_{\rm rec}^{1/2}$ is SPD and not bare $L_{\rm acc}$ in Euclidean coordinates.
Generalized spectrum	PASS	$M_{\rm rec}^{-1/2} K_{\rm rec}\, M_{\rm rec}^{-1/2}$ recovers the $L_{\rm acc}$ spectrum to 1.48×10^{-12}.
Matched EM cone	PASS	$c_{\rm em} = c_* = 3.1256671980$; $g_{\rm eff}$ cancels from the record cone.
Maxwell stationarity	PASS	$g_{\rm eff}^{-2} K_{\rm rec} A = J$ holds to numerical roundoff.
Bare α^{-1}	STRUCTURAL	$42\pi + 16/\pi = 137.0398496297$.
Continuum correction	LEGACY	$42\pi + 16/\pi - 3/(8\pi^4) = 137.0359998864$.
Naive universal-$M_{\rm rec}$ with bare $K_{\rm rec}$	FAIL	Gives 136.9685945885 because first-mode loading is counted without matching stiffness.

Item	Result	Meaning
Paired M_{rec}-aware finite correction	PASS (toy)	Gives 137.0359208260, restoring the finite continuum correction under the universal pair.
Charged-lepton ladder	STRUCTURAL ONLY	$m_\mu/m_e = 206.5538812391$ and $m_\tau/m_e = 3495.4159810640$ in the paired finite audit.

13.7 Status of the paired audit

The paired construction is the correct finite-toy consistency move for the universal record pair. The framework should not make M_{rec} universal while leaving K_{rec} bare. The physical object is the generalized pair $(K_{\text{rec}}, M_{\text{rec}})$, and every sector inherits the same pair up to sector-specific scalar coupling factors such as g_{eff}^{-2} in the electromagnetic amplitude.

The construction should not be oversold. The metric-compatible lift

$$K_{\text{rec}} \;=\; M_{\text{rec}}^{1/2}\, L_{\text{acc}}\, M_{\text{rec}}^{1/2}$$

is one realization of the substrate-side weaker compatibility condition $\text{spec}(M_{\text{rec}}^{-1/2} K_{\text{rec}}\, M_{\text{rec}}^{-1/2}) \to \text{spec}(G_{\text{acc}})$ stated in Section 2. It repairs the finite-toy inconsistency and preserves the charged-sector audit, but a paper-level derivation must explain why the universal stiffness must take this form, or must derive an equivalent paired structure from a substrate variational principle. The technical problem is therefore sharper: derive the universal record pair $(K_{\text{rec}}, M_{\text{rec}})$ from the substrate so that classical motion, Maxwell stationarity, finite record cones, horizon-like susceptibility growth, quantum/access non-separation, and the charged-sector access-polarization and lepton-ladder diagnostics all use the same generalized stiffness/susceptibility geometry.

The role of this audit in the fine-structure diagnostic of Section 18 is consistency-side, not derivation-side. It rules out one nearby alternative — universalizing only M_{rec} while leaving K_{rec} bare — as failing the finite consistency test, and confirms that the paired metric-compatible lift restores the continuum correction value. It does not resolve the ansatz $\alpha_{\text{junction}}^{-1} = 4\pi\, T_{\text{bare}}$, the unnormalized-versus-canonical Y-trace question in $T_{\text{bare}} = 21/2$, the substrate motivation for the Dirichlet boundary condition, or the lepton-ladder precision miss. Those remain in the remaining-problems list of Section 18.

14 Relation to neighboring frameworks

The framework sits near several established approaches. Its proposed contribution is a different bookkeeping discipline: substrate-side structure (carried by the paired $(K_{\text{rec}}, M_{\text{rec}})$ operator) and observer-side reconstruction are kept explicitly separate, and the finite validations track which assumptions are substrate axioms, which are observer ansätze, and which results are constructed consequences.

Page–Wootters. Page–Wootters models [20, 21, 22] recover dynamics from correlations inside a globally stationary quantum state by using an internal clock subsystem. The present framework is close in spirit but does not identify the observer merely with a clock. It gives the observer a record algebra $M_\lambda \subseteq A_\lambda$ and derives the access order from record recoverability through the paired generator $G_{\text{acc}} = M_{\text{rec}}^{-1/2} K_{\text{rec}} M_{\text{rec}}^{-1/2}$. This record-recoverability order is the main distinguishing bookkeeping move.

Relational quantum mechanics. Relational QM [23] emphasizes that states are relative to observers or systems. The present framework accepts observer-relative state assignment but adds a fixed location map: stable separability, the paired disturbance bookkeeping, stiffness, and connection data are substrate-side; time, collapse, motion, field evolution, and Lorentz kinematics are observer-side.

Connes–Rovelli thermal time. The thermal-time hypothesis [24] extracts a time flow from the modular automorphism group of a state on a von Neumann algebra. The present framework differs in aim: it treats experienced time as record-recoverability order, not as a canonical modular flow. The bandwidth radius and access speed are induced by the lowest generalized eigenvalue of the paired G_{acc}.

Jacobson thermodynamics of spacetime. Jacobson [25, 26] derives the Einstein equation as an equation of state from the Clausius relation applied to local causal horizons. The present paper agrees with the structural moral that gravity-like dynamics may be thermodynamic rather than microscopic. It differs in primitive data: Jacobson's local horizons are spacetime objects; the present framework's cuts are algebraic conditional expectations selected by modular stability through $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$. Thermodynamics is recovered only from substrate–observer coupling: $\delta Q_R = \Theta_R \delta S_R$, where δQ_R is paired-disturbance-cost variation across a cut and δS_R is accessible record-entropy variation through the same cut.

Decoherent histories, causal sets, and tensor networks. The decoherent-histories program [27, 28, 29] also locates the operational arrow in record-bearing structure, but stays inside ordinary quantum mechanics on a spacetime background; here the substrate is tenseless and the record order is intrinsic to the observer junction. Causal set theory [30] replaces spacetime with a discrete partial order; the present framework keeps the algebraic side continuous and lets order arise from record inclusion rather than from a primitive causal order. Tensor-network and emergent-geometry constructions [31, 32] build spacetime from entanglement structure; in the present language those constructions can be read as observer-side reconstruction of access geometry from a substrate-side correlation pattern, with the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ supplying the cone speed.

Crossed-product observer programs. Recent work on observer algebras and the crossed product in semiclassical gravity [17, 18, 19] shows how Type III algebras become Type II once an observer with a clock is included. The bookkeeping here is structurally parallel — a tracial substrate with a non-tracial observer cut — but starts from the substrate side and treats the Type II/III question as a junction question rather than a gravity-input question.

The value of the framework is diagnostic: it marks which effective laws are substrate-only, which are observer-only, and which are recovered only at their junction. Newtonian mechanics is recovered as least-disturbance record continuation; Maxwell-type equations as stationary holonomy disturbance with the matched pair $K^{\mathrm{em}}_{\mathrm{rec}} = g_{\mathrm{eff}}^{-2} K_{\mathrm{rec}}$, $M^{\mathrm{em}}_{\mathrm{rec}} = g_{\mathrm{eff}}^{-2} M_{\mathrm{rec}}$; weak-field gravity as stationary accessible connection response to paired regional disturbance; and Lorentz kinematics as a conditional invariant-access-cone recovery.

15 Open questions

15.1 Type II_1 to Type III_1 observer junction

The central structural problem is whether local entropy-maximizing observer states on increasing access cuts, constrained by stable records and denied the global trace, can converge to Type III_1-like modular algebras. The first target is a rigorous version of the modular-density diagnostic:

construct a sequence (A_n, ω_n) over finite trace approximants such that the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ disturbance normal form remains stable while the modular ratio groups of ω_n become dense in $\mathbb{R}$.

The framework leaves several questions open.

1. *Continuum and refinement.* Implement algebraic shell growth and subalgebra joins on genuine von Neumann subalgebra systems without embedded coordinate assumptions.

2. *Justifying the primitive principles.* Derive the paired master regional disturbance functional $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$, its admissible readout maps, and the structural decomposition $K_{\mathrm{rec}} = K_0 + \sum_\bullet \kappa_\bullet K_\bullet$, $M_{\mathrm{rec}} = M_0 + \sum_\bullet \eta_\bullet M_\bullet$ from a more primitive operator-algebraic condition.

3. *Operator-algebraic implementation.* A genuine implementation on a von Neumann algebra is the natural next mathematical step.

4. *Observer reconstruction at scale.* A larger-scale demonstration with realistic record-bearing dynamics is required.

5. *Observer-effective quantum reconstruction.* A larger operator-algebraic reconstruction would need non-commuting accessible algebras, POVMs, entanglement across cuts, decohered record subalgebras, and a principled account of Born weight selection.

6. *Bandwidth-derived Lorentz observer kinematics.* A stronger account would prove convergence of the generalized spectrum $K_{\mathrm{rec}} v = c^2 M_{\mathrm{rec}} v$ from the Type II_1 trace geometry and derive Lorentz transformations between different observer cuts.

7. *Relativistic extension.* A genuine relativistic extension requires tensor stress-energy with the right signature, a metric or richer connection sector, a variational principle, and an account of diffeomorphism redundancy.

8. *Refinement-program hardening.* Replace embedded-coordinate benchmarks with a substrate-intrinsic refinement category; construct transfer operators from algebraic conditional expectations; extend metric action beyond a quadratic positive-definite perturbative model; couple metric and gauge fields; run Level-1 non-Abelian validations.

9. *Comparison with alternative framings.* A systematic comparison with relational QM, Page–Wootters, thermal time, decoherent histories, causal sets, and tensor-network geometries is open work.

10. *Modular thermodynamics.* Make $\delta Q_R = \Theta_R \delta S_R$ intrinsic using modular relative entropy rather than finite approximants.

11. *Type* III_1 *lift.* Lift the shared-generator composition-independence result from Type II_1 trace geometry to a Type III_1 modular setting.

15.2 Einstein-reduction proof obligations

The continuum Einstein-translation theorem schema reduces the gravity problem to four precise proof obligations. First, the generalized record operator G_{acc} must lie in a Laplace-type spectral universality class with nonzero scalar-curvature coefficient. Second, admissible metric variations must be lifted from algebraic automorphisms. Third, the observer access-cut and recovery-loss functional must supply the boundary term. Fourth, the coefficient of the scalar-curvature term must be normalized against $c_*^2 = \lambda_{\min}(G_{\mathrm{acc}})$ so that $\kappa_{\mathrm{eff}} = 8\pi G_{\mathrm{obs}}/c_*^4$.

15.3 Horizon-cut radiation

Compute $\Theta_{\partial B}$ from modular data; derive or falsify $\Theta_{\partial B} \propto \kappa_{\rm rec}$; define a finite model exhibiting a Page-like recoverability transition.

15.4 Cosmological-constant dictionary entry

A cosmological-constant-like term would be placed as a global paired disturbance baseline or vacuum offset, not as localized matter disturbance. The present work does not derive such a term; it only identifies its placement.

16 Conclusion

The thesis is organizational, not predictive. Two layers separate cleanly: a tenseless algebraic substrate carries stable relational structure through the paired $(K_{\rm rec}, M_{\rm rec})$ bookkeeping, and a memory-bearing observer carries record-ordered access. The compressed substrate object is the paired regional disturbance: the universal record stiffness $K_{\rm rec}$ and the universal record susceptibility $M_{\rm rec}$, with the observable record cone governed by the generalized record operator $G_{\rm acc} = M_{\rm rec}^{-1/2} K_{\rm rec} M_{\rm rec}^{-1/2}$. Locality, non-separation, persistence, support-stiffness, gravity-like response, and gauge connection are not treated as separate primitive disturbance sectors; they are readouts or stationarity conditions of this paired disturbance under different probes. Experienced time, motion, measurement update, and field evolution remain observer-side effects of stable record access.

The operational replacement for primitive time is the inclusion index of stable records. A modular time scale appears when the local entropy-maximizing observer state on the recoverable algebra is faithful and nontracial. This keeps the substrate tenseless while giving the observer layer enough structure to support thermal flow, finite access cones, and relativistic kinematics as reconstruction targets.

The central gain of the compressed formulation is that entropy, inertia, and gravity-like response no longer need to be engineered as parallel functionals. Entropy is scalar unrecoverability through $M_{\rm rec}$. Inertia is the algebraic support-deformation Hessian through $K_{\rm rec}$. Gravity-like response is the connection variation of the same paired disturbance. The light-speed limit is $c_*^2 = \lambda_{\rm min}(G_{\rm acc})$. The electromagnetic sector matches the same paired structure, $(K_{\rm rec}^{\rm em}, M_{\rm rec}^{\rm em}) = g_{\rm eff}^{-2}(K_{\rm rec}, M_{\rm rec})$, ensuring that the coupling controls amplitude without altering the universal record cone.

The Type II_1 normal-form theorem remains the substrate anchor. Under explicit covariance and isotropy assumptions, local conditional-expectation defects have one canonical quadratic trace geometry, and the paired $(K_{\rm rec}, M_{\rm rec})$ structure inherits the same canonical trace.

The observer junction remains essential. A purely tracial substrate has no experienced time, no nontrivial thermal flow, and no primitive light speed. Nontracial observer states and scaling limits supply modular time, finite recoverability cones, temperature-like parameters, and Type III-like behavior. The black-hole reading uses the same separation: horizon entropy, exterior information loss, and radiation-like flux are observer-access readouts of large boundary paired disturbance, while the full substrate description is not destroyed.

The strongest future direction is to replace the remaining constructive choices with primitive principles. The paired master disturbance functional, admissible regional category, stiffness and susceptibility operators with their structural decompositions, source readouts, and refinement protocols are still specified rather than uniquely derived. The finite QEC examples show how conditional-expectation defect diagnoses Petz recoverability and channel recovery loss in concrete finite trace settings consistent with the paired framework. Until the paired master disturbance and observer-access category are derived from sharper axioms, this paper should be read as a careful foundations and constructive-reconstruction essay with finite consistency and refinement validations, not as an empirical theory.

17 Validation material outside the main argument

Companion validation protocols include a six-region Level-0 construction, Newtonian and Lorentz observer finite-model models, sourced $U(1)$ solves through the matched pair $(K_{\rm rec}^{\rm em}, M_{\rm rec}^{\rm em})$, weak-field scalar solves, coordinate-graph refinement tests, topological stress tests, and finite non-Abelian pilot scripts. The sourced $U(1)$ solve is summarized in the main Maxwell-type section only as a stationarity benchmark for the holonomy readout. The remaining calculations are useful as consistency checks, but they are not premises of the foundation argument.

The main claim is therefore not that finite scripts reproduce selected formulas. The main claim is that conditional-expectation defect supplies the local geometry of a paired master regional disturbance $(K_{\rm rec}, M_{\rm rec})$; entropy, recoverability loss, inertial support stiffness, gravity-like connection response, and gauge stationarity are readouts of that one paired disturbance through observer-access cuts.

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18 Internal charged sector: identification, minimality, and observer-junction diagnostics

The framework's substrate layer fixes a tenseless algebraic totality with paired regional disturbance $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$; the observer layer fixes record-ordered cuts. Up to this point the internal automorphism sector of the substrate has been treated as additional declared structure. This section asks how far the framework can constrain that sector by substrate-style principles, what charged-sector structure follows, and what numerical diagnostics emerge at the observer junction through the paired bookkeeping.

The result is not a derivation of the Standard Model. It is a chain of conditional consequences. If physical charge is identified with the minimal observer-stable internal holonomy obstruction visible to admissible cuts, and if the access-mixing structure is built from the same observer cut and the same triple-overlap obstruction by the smallest trace-norm preserving recombinations, then the minimal internal algebra is fixed, the minimal chiral content is fixed, the hypercharge values are fixed by recoverability-obstruction cancellation, the trace ratio $\mathrm{Tr}(Y^2)/\,\mathrm{Tr}(T_3^2) = 5/3$ and the substrate-scale weak angle $\sin^2\theta_W = 3/8$ follow, and a numerical fine-structure diagnostic and a charged-lepton ladder become available through the paired $(K_{\mathrm{rec}}, M_{\mathrm{rec}})$ structure.

The guiding methodological rule throughout is that no measured value of α, m_e, m_μ, m_τ, or m_p is used to choose a coefficient. The electron mass enters only as a normalization unit when reporting dimensionless lepton ratios.

18.1 Charge as minimal observer-stable internal holonomy

A candidate charged sector is taken to be a nontrivial, observer-stable, internal holonomy label of a persistent regional defect. Four requirements are imposed:

- *internality*: not a displacement in substrate time or observer-effective spacetime;
- *record stability*: survives admissible observer cuts through the paired barrier;
- *relationality*: compares internal frames between access algebras;
- *non-degeneracy*: not a trivial identity or backtracking inverse.

Let $A_i, A_j, A_k \subset W$ be local access algebras and $U_{ij} : A_j \to A_i$ admissible internal transition maps. A one-frame loop $U_{ii} = 1$ carries no charged distinction. A two-frame loop $U_{ij}U_{ji} = 1$ tests reversibility. The first nondegenerate compatibility obstruction is a triple comparison:

$$\Omega_{ijk} = U_{ij}U_{jk}U_{ki}.$$

Under the *primitive scalar-isotropic charged-cell assumption* (the observer-readable part of the primitive charged obstruction lies in a central Abelian phase sector; the three edge transports are equivalent under cell symmetry; the lowest-disturbance charged readout is the scalar component of the obstruction), each edge carries the same central phase z:

$$U_{ij} = U_{jk} = U_{ki} = z, \qquad \Omega_{ijk} = z^3.$$

Record-stable closure requires $z^3 = 1$, giving three solutions: $z_0 = 1$, $z_1 = e^{2\pi i/3}$, $z_2 = e^{4\pi i/3}$. The primitive charged sector has exactly three stable branches D_0, D_1, D_2.

Higher N-frame loops decompose into triple-overlap pieces; thus $N \geq 4$ closures may represent composite or excited charged structures but do not define new primitive elementary charged sectors.

Proposition 18.1 (Primitive isotropic triple-overlap charged sector)**.** *Let a charged sector be defined as a nontrivial, observer-stable, internal holonomy label of a persistent regional defect. Assume the substrate admits local access algebras with invertible pairwise internal transition maps U_{ij}. Then the first nondegenerate charged obstruction is $\Omega_{ijk} = U_{ij}U_{jk}U_{ki}$. Under the primitive scalar-isotropic charged-cell assumption, record closure gives $z^3 = 1$ and the primitive charged sector has exactly three stable branches.*

18.2 From discrete branches to continuous mixing

The triple-overlap closure delivers three discrete charged branches. The passage to the familiar non-Abelian factors requires two further moves grounded in substrate/access primitives.

Two-state access mixing as cut/complement. The substrate-derived two-state structure is the cut/complement decomposition at a fixed access label. The observer cut $E_\lambda : W \to A_\lambda$ defines, given a local charged disturbance D, two primitive components $D_\parallel = E_\lambda(D)$, $D_\perp = D - E_\lambda(D)$, so the minimal access-discrimination object is the pair

$$\Psi_D = \begin{pmatrix} D_\parallel \\ D_\perp \end{pmatrix}.$$

The trace-quadratic disturbance norm (weighted by the paired $(K_{\text{rec}}, M_{\text{rec}})$ at the same cut) is

$$\|\Psi_D\|_\tau^2 = \tau(D_\parallel^* D_\parallel) + \tau(D_\perp^* D_\perp).$$

Admissible internal recombinations preserving this norm form a $U(2)$-type group on the two-component access space. Removing the central Abelian phase (already counted by the holonomy sector) leaves

$$SU(2).$$

Continuous mixing of the three primitive branches. The triple-overlap closure produces a three-dimensional branch space $\mathcal{H}_3 = \text{span}\{D_0, D_1, D_2\}$. Admissible internal recombinations preserving the trace-quadratic branch norm act by $U(3)$. Removing the separately counted central $U(1)$ phase leaves

$$SU(3).$$

The corrected chain is: primitive triple-overlap obstruction $\Rightarrow$ 3 branches $\Rightarrow U(3) \Rightarrow SU(3)$ after removing $U(1)$. The discrete closure $z^3 = 1$ supplies the primitive three-dimensional branch space; the continuous $SU(3)$-like factor is the minimal trace-norm preserving non-Abelian mixing.

18.3 Minimal charged-sector selection principle

Principle 18.2 (Minimal charged-sector selection)**.** *The physical charged sector is the smallest nontrivial observer-stable internal automorphism sector that simultaneously satisfies:*

1. *a central Abelian phase readout;*
2. *minimal non-Abelian two-state access mixing;*

3. *minimal non-Abelian three-frame holonomy-index structure;*

4. *chiral doublet/singlet access splitting;*

5. *cancellation of global recoverability obstructions.*

A pure $U(1)$ fails (2), (3), (4). $SU(2) \times U(1)$ fails (3). $SU(3) \times U(1)$ fails (2), (4). $SU(2) \times SU(2) \times U(1)$ fails (3). $SU(5)$ or $SU(4) \times SU(2) \times U(1)$ contain extra branches; they fail minimality.

The minimal product is therefore the Standard Model gauge group [49, 50, 51]

$$SU(3) \times SU(2) \times U(1).$$

Factor	Framework origin
Abelian charge	Central phase of primitive internal holonomy obstruction
Two-state access mixing	Cut/complement pair $(D_\parallel, D_\perp)$ at one cut
Three-branch holonomy mixing	Primitive triple-overlap obstruction; trace-norm preserving mixing of three

Two constraints — chiral doublet/singlet access splitting and cancellation of global recoverability obstructions — are imported as structural requirements in the present treatment.

18.4 Minimal chiral content and hypercharge fixing

Once the minimal internal algebra is $SU(3) \times SU(2) \times U(1)$, the minimal chiral content is selected by representation theory:

$$Q_L : (3,2), \quad u_R : (3,1), \quad d_R : (3,1), \quad L_L : (1,2), \quad e_R : (1,1),$$

up to neutral singlets. Let Y_Q, Y_u, Y_d, Y_L, Y_e be unknown. The framework analogue of anomaly cancellation [54, 55, 56] reads:

$$\begin{aligned} SU(3)^2U(1) : &\quad 2Y_Q - Y_u - Y_d = 0, \\ SU(2)^2U(1) : &\quad 3Y_Q + Y_L = 0, \\ U(1)^3 : &\quad 6Y_Q^3 - 3Y_u^3 - 3Y_d^3 + 2Y_L^3 - Y_e^3 = 0. \end{aligned}$$

With $Q_{\text{em}} = T_3 + Y$ and normalization $Y_e = -1$, the minimal rational solution is

$$Y_Q = \tfrac{1}{6}, \quad Y_u = \tfrac{2}{3}, \quad Y_d = -\tfrac{1}{3}, \quad Y_L = -\tfrac{1}{2}, \quad Y_e = -1.$$

The one-generation Abelian trace is $T_{\text{gen}} = \operatorname{Tr}(Y^2) = 6(\frac{1}{6})^2 + 3(\frac{2}{3})^2 + 3(-\frac{1}{3})^2 + 2(-\frac{1}{2})^2 + (-1)^2 = 10/3$. The one-generation weak trace is $\operatorname{Tr}(T_3^2) = 3 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = 2$.

18.5 Hypercharge normalization and conditional weak angle

The ratio is $\operatorname{Tr}(Y^2)/\operatorname{Tr}(T_3^2) = (10/3)/2 = 5/3$. In partition-strain language $\operatorname{Tr}(Y^2)$ is the Abelian internal holonomy stiffness trace through K_{rec} and $\operatorname{Tr}(T_3^2)$ is the weak doublet stiffness trace. Trace-normalized stiffness equality defines the canonically normalized Abelian generator

$$Y_{\text{can}} = \sqrt{\tfrac{3}{5}}\, Y, \qquad \operatorname{Tr}(Y_{\text{can}}^2) = \operatorname{Tr}(T_3^2),$$

equivalently $g_1^2 = (5/3) g_Y^2$. This is the usual 5/3 hypercharge normalization, obtained as a trace-ratio consequence of the minimal one-generation charged content rather than as an $SU(5)$ [52, 53] embedding postulate.

If the canonically normalized primitive Abelian and weak sectors share the same leading paired stiffness coefficient through $K_{\rm rec}$, $g_1 = g_2$ at the substrate normalization scale, then

$$\sin^2\theta_W = \frac{g_Y^2}{g_2^2+g_Y^2} = \frac{3/5}{1+3/5} = \frac{3}{8}.$$

This is the unification-scale value, not the observed low-energy weak angle. The 3/8 value is a structural consistency check on the trace logic, not a low-energy prediction.

18.6 Conditional identification with the Maxwell readout and the lepton ladder

Proposition 18.3 (Conditional charged-sector identification)**.** *Assume the observer-effective charged sector is the minimal internal sector satisfying: internality, observer-record stability, relationality, central Abelian phase readout, entry into the regional paired disturbance functional through the same internal connection whose stationary variation defines the Abelian observer field readout, and use of its stable branches as the charged identity sectors for the record-preserving strain ladder. Then the primitive charged obstruction is Ω_{ijk}. Isotropic Abelian closure gives $z^3 = 1$, hence three stable charged branches. The same internal connection produces the Maxwell-type observer readout through stationarity of $\mathcal{K}_R$ with the matched pair $(K^{\rm em}_{\rm rec}, M^{\rm em}_{\rm rec}) = g_{\rm eff}^{-2}(K_{\rm rec}, M_{\rm rec})$, and the same three branches supply the charged strain ladder used in the lepton diagnostic.*

Proof sketch. The triple-overlap argument supplies the three branches. For the Maxwell readout, perturb the internal Abelian connection by A:

$$\mathcal{K}_R(\nabla + A) = \mathcal{K}_R(\nabla) + \tfrac{1}{4g_{\rm eff}^2}\langle F_A, F_A\rangle_{\tau,R} + \langle A, J\rangle + O(A^3),$$

where $F_A = dA$. Stationarity $\delta_A \mathcal{K}_R = 0$ gives $\delta F_A = g_{\rm eff}^2 J$. Combined with $dF = 0$:

$$dF = 0, \qquad \delta F = g_{\rm eff}^2 J.$$

The matched pair $(K^{\rm em}_{\rm rec}, M^{\rm em}_{\rm rec})$ ensures the electromagnetic record cone equals the universal c_*. The lepton ladder follows from using the three stable branches as charged identity sectors and the bi-Laplacian stiffness $E_k = k^4$, $S_n = \sum_{k=1}^n k^4$. □

18.7 Fine-structure diagnostic from access polarization

The Maxwell-type sector identified above gives an Abelian coupling fixed by the paired regional disturbance expansion,

$$\alpha_{\rm sub} = \frac{g_{\rm sub}^2}{4\pi}, \qquad \alpha_{\rm sub}^{-1} = 4\pi g_{\rm sub}^{-2}.$$

The diagnostic question is whether the observer-effective inverse coupling can be obtained from a fixed access-stiffness calculation. The calculation is organized through the paired $(K_{\rm rec}, M_{\rm rec})$ structure.

Access-polarization operator and first-channel dominance. Normalize the observer-access interval to $u \in [0,1]$ and let $L_{\rm acc} = K_{\rm acc}$ be the access Laplacian with Dirichlet boundary conditions. Its normalized modes are $\psi_n(u) = \sqrt{2}\sin(n\pi u)$, $L_{\rm acc}\psi_n = (n\pi)^2\psi_n$. The bi-Laplacian susceptibility operator entering the $M_{\rm rec}^{-1}$ readout is

$$L_{\rm acc}^{-2}\psi_n = \frac{1}{(n\pi)^4}\psi_n.$$

The access-polarization correction is the trace of a charged access susceptibility operator,

$$\Pi_{\text{access}} = \mathrm{Tr}_{\text{acc}}\big(P_{\text{ch}} L_{\text{acc}}^{-2}\big),$$

where P_{ch} projects onto the charged part of the recoverable access channel. If the recoverable Abelian probe is uniform, the first mode dominates because the next nonzero mode (Dirichlet $n = 3$) is suppressed by $3^{-4} = 1/81$. The first-channel approximation is therefore $P_{\text{acc}} \simeq P_1 := |\psi_1\rangle\langle\psi_1|$, as the leading term of the inverse bi-Laplacian access susceptibility.

The first-channel amplitude of the uniform probe is $a_1 = \int_0^1 \sin(\pi u)du = 2/\pi$, so the first record-projection contribution to the bare junction stiffness is

$$\Delta_{\text{obs}} = a_1^2 = \frac{4}{\pi^2}.$$

Single trace-normalized primitive stiffness. Once primitive internal generators are normalized by the substrate trace, the leading quadratic stiffness through K_{rec} is shared by primitive weak and Abelian access channels. This is the internal analogue of the Type II_1 shared-normal-form logic: after canonical trace normalization, there is one leading trace-quadratic stiffness coefficient for primitive low-disturbance internal generators. Thus

$$\mathrm{Tr}(Y_{\text{can}}^2) = \mathrm{Tr}(T_3^2), \qquad g_1 = g_2$$

at the substrate/junction normalization level.

Charged-sector trace and the charged access projector. The minimal charged content gives $\mathrm{Tr}(Y^2)/\,\mathrm{Tr}(T_3^2) = 5/3$. With $g_1 = g_2$, the Abelian fraction of the primitive weak/Abelian access channel is

$$w_Y = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{3}{8}.$$

The charged projector on the first recoverable access channel is therefore $P_{\text{ch}} = w_Y P_1 = (3/8)P_1$. Substitution gives

$$\Pi_{\text{access}} = \frac{3}{8}\,\mathrm{Tr}_{\text{acc}}(P_1 L_{\text{acc}}^{-2}) = \frac{3}{8\pi^4}.$$

Bare junction stiffness. The bare Abelian stiffness is the charge-square trace over the minimal chiral content. The one-generation trace is $T_{\text{gen}} = 10/3$. If the three triple-overlap branches are interpreted as three generation copies of the minimal chiral charged multiplet, the three-generation fermionic trace is $T_{\text{fermions}} = 3T_{\text{gen}} = 10$. A minimal access-stabilizing doublet $H = (H^+, H^0) : (1, 2)$, $y_H = 1/2$, contributes $T_H = 2(1/2)^2 = 1/2$. The bare trace is

$$T_{\text{bare}} = T_{\text{fermions}} + T_H = 10 + \tfrac{1}{2} = \tfrac{21}{2} = 10.5.$$

With the first record-projection correction, $T_{\text{junction}} = T_{\text{bare}} + \Delta_{\text{obs}} = 10.5 + 4/\pi^2$, and

$$\alpha_{\text{junction}}^{-1} = 4\pi T_{\text{junction}} = 42\pi + \frac{16}{\pi} = 137.0398496\ldots$$

This is a bare observer-junction value, before charged access polarization through M_{rec} dresses the Abelian channel.

Corrected diagnostic value. The observer-effective inverse coupling is read as

$$\alpha_{\text{eff}}^{-1} = \alpha_{\text{junction}}^{-1} - \Pi_{\text{access}}.$$

Combining the bare trace, first record-projection correction, and first access-polarization correction:

$$\alpha_{\text{eff}}^{-1} = 4\pi\left(10.5 + \frac{4}{\pi^2}\right) - \frac{3}{8\pi^4} = 42\pi + \frac{16}{\pi} - \frac{3}{8\pi^4} = 137.035999886\ldots$$

The observed inverse fine-structure constant is $\alpha^{-1} \approx 137.035999177$ [57, 58, 59, 60], so the remaining diagnostic difference is $\Delta\alpha^{-1} \approx 7.09 \times 10^{-7}$, relative error $\approx 5.2 \times 10^{-9}$.

The sign should not be read as ordinary high-scale-to-low-scale QED running. Here α_{junction} is a bare observer-junction stiffness, not a grand-unification-scale coupling. The subtraction represents access dressing through the paired structure: charged record modes make the observer-accessible Abelian channel slightly more compliant and lower the inverse stiffness.

Consistency and ablation status. The factor 3/8 in P_{ch} is a trace-normalized weak/Abelian access fraction, not the measured low-energy weak angle. The π^4 is the first bi-Laplacian eigenvalue of the normalized Dirichlet access interval. The correction needs an ablation test before it carries evidential weight: enumerate nearby symbolic alternatives built from quantities already present $(3/8, 5/3, 3/2, 10/3, 1/2$, powers of $\pi)$ and check whether $3/(8\pi^4)$ is isolated.

18.8 Charged-lepton ladder

The three stable triple-overlap branches D_0, D_1, D_2 are now used as charged identity sectors of the record-preserving access ladder

$$D_0 \xrightarrow{A_1} D_1 \xrightarrow{A_2} D_2.$$

The observer cannot access D_2 while erasing the record of the path through D_1, so the activated set is the prefix set $A_n = \{1, 2, \ldots, n\}$, and the strain readout is cumulative: $S_n = \sum_{k=1}^{n} E_k$. The single-mode energy uses the bi-Laplacian stiffness $L\psi_k = k^2\psi_k$, $L^2\psi_k = k^4\psi_k$, so $E_k = k^4$:

$$S_n = \sum_{k=1}^{n} k^4, \qquad S_0 = 0,\ S_1 = 1,\ S_2 = 17.$$

The ladder mobility-suppression coefficient is a counted ratio of charged holonomy branches to observer access boundaries:

$$C_{\text{ladder}} = \frac{N_{\text{hol}}}{N_{\text{bdry}}} = \frac{3}{2}.$$

Using the corrected observer-effective fine-structure diagnostic,

$$B_{\text{eff}} = C_{\text{ladder}}\alpha_{\text{eff}}^{-1} = \tfrac{3}{2}\alpha_{\text{eff}}^{-1} = 205.5539998\ldots,$$

and the charged-lepton ladder readout is

$$\frac{m_n}{m_e} = 1 + \tfrac{3}{2}\alpha_{\text{eff}}^{-1}\sum_{k=1}^{n} k^4, \qquad n = 0, 1, 2.$$

The three values are $m_0/m_e = 1$, $m_1/m_e \approx 206.5540$, $m_2/m_e \approx 3495.42$. Observed [61]: $m_\mu/m_e \approx 206.7683$, $m_\tau/m_e \approx 3477.15$. Relative errors: -0.10% and $+0.53\%$. These are catastrophic at experimental precision; the ladder formula reproduces the structural form but the underlying mass mechanism has not been correctly identified.

18.9 Charge structure consequences

Without further parameters, the minimal chiral content gives, via $Q_{\text{em}} = T_3 + Y$:

$$Q_u = +\tfrac{2}{3}, \quad Q_d = -\tfrac{1}{3}, \quad Q_e = -1, \quad Q_\nu = 0.$$

Valence accounting: $Q_p = +1$, $Q_n = 0$, $Q_H = Q_p + Q_e = 0$. These are charge-algebra consequences of the minimal chiral content.

18.10 What is not recovered

Proton-electron mass ratio. The proton is a confined composite, not a charged ladder mode. Its mass requires a baryonic confinement/binding readout not constructed in the framework.

Low-energy weak mixing angle. $\sin^2\theta_W = 3/8$ is the substrate-scale unification value, not the measured low-energy value.

Generation count. Whether the same $\mathbb{Z}_3$ supplies both color and generations is not resolved.

Chiral access splitting. The cut/complement argument supplies $SU(2)$ but not chirality; chirality is asserted as a primitive constraint.

Recoverability-obstruction equation form. The form of the cubic-trace constraints is imported as the framework analogue of anomaly cancellation.

Fine-structure and lepton-ladder values at experimental precision. The access-polarization correction brings the fine-structure diagnostic to the 10^{-9} relative-error level, but the first-channel truncation, charged projector, boundary normalization, and nearby symbolic alternatives still require substrate-level justification or ablation. The charged-lepton ladder remains at the 10^{-3} relative-error level.

18.11 Status and remaining problems

Item	Status
Primitive charged holonomy	Conditionally triple-overlap governed.
Three charged branches	From triple-overlap closure plus primitive scalar-isotropic charged-cell assumption.
Two-state access mixing	$SU(2)$ from cut/complement modulo central phase.
Three-branch holonomy mixing	$SU(3)$ as continuous trace-norm preserving mixing modulo central phase.
Minimal charged algebra	$SU(3) \times SU(2) \times U(1)$ from selection principle.
Minimal chiral content	Determined up to neutral singlets.
Hypercharge values	From recoverability-obstruction cancellation.
Hypercharge normalization 5/3	From $\mathrm{Tr}(Y^2)/\,\mathrm{Tr}(T_3^2)$.
Weak angle 3/8	Substrate-scale value only, conditional on $g_1 = g_2$.
Identification with Maxwell sector	Conditional theorem with matched pair $(K^{\text{em}}_{\text{rec}}, M^{\text{em}}_{\text{rec}})$.
Charged-lepton ladder	Structural diagnostic, not at experimental precision.

Item	Status
Fine-structure diagnostic	Bare $\alpha^{-1}_{\text{junction}} = 42\pi + 16/\pi$; with first access-polarization correction, $\alpha^{-1}_{\text{eff}} = 42\pi + 16/\pi - 3/(8\pi^4)$. Reverse-engineered structural diagnostic pending derivation of the polarization operator.
Electric charge pattern	Charge-algebra consequence.
Proton-electron mass ratio	Not recovered; baryonic sector required.
Three generations	$\mathbb{Z}_3$ double duty not resolved.
Chiral access splitting	Imported as primitive constraint.
Recoverability-obstruction form	Imported as framework analogue of anomaly cancellation.

The framework has not proved the physical charged sector from the substrate alone. It has localized the remaining assumptions inside a single explicit principle: the physical charged sector is the minimal observer-stable internal automorphism sector satisfying central Abelian phase readout, cut/complement two-state access mixing, three-branch triple-overlap holonomy mixing, chiral doublet/singlet splitting, and global recoverability-obstruction cancellation. Under this principle the minimal charged algebra, the hypercharge pattern, the 5/3 trace normalization, the conditional unification-scale weak angle, the fine-structure diagnostic, its access-polarization correction, and the charged-lepton ladder all follow as one conditional chain through the paired $(K_{\text{rec}}, M_{\text{rec}})$ bookkeeping rather than as independent fits.

The remaining substrate-level work consists of five explicit problems. First, derive the primitive scalar-isotropic charged-cell assumption from the substrate. Second, derive the chiral access splitting from substrate/access primitives. Third, derive the specific form and multiplicities of the recoverability-obstruction equations. Fourth, distinguish the $\mathbb{Z}_3$ producing color from the $\mathbb{Z}_3$ producing generations. Fifth, derive the access-polarization operator assumptions from substrate principles; these choices must be fixed or ablated against nearby symbolic alternatives. Together these problems define the remaining technical program for the charged-sector layer.

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